Math 101-151-Class Test II

Name: ID Serial:

Show all your work. No credits for answers not supported by work

1) Find $\frac{dy}{dx}$ for the following

a) $y = \frac{e^{x+1} - e^x}{e^x}$

$y = \frac{e^{x+1} - e^x}{e^x} = \frac{e^{x+1}}{e^x} - \frac{e^x}{e^x} = e - 1$ *1pt.*

$\frac{dy}{dx} = 0$ *1pt.*

b) $y = \frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{1}{1+\left(\frac{x+1}{3}\right)^2}\right)\left(\frac{1}{3}\right)$ *1pt.*

$= \frac{1}{9+(x+1)^2}$ *1pt.*

c) $y = \log_3(x^2 + 1)$

$\frac{dy}{dx} = \frac{1}{1n\ 3}\left(\frac{2x}{x^2+1}\right) = \frac{2x}{(x^2+1)\ln 3}$ *2ps.*

2) If $f(x) = \log_{10} \sqrt[3]{\frac{x^2}{(x-1)^4}}$, then find $f'(2)$

$f(x) = \log_{10} \sqrt[3]{\frac{x^2}{(x-1)^4}} = \frac{1}{3}(\log_{10} x^2 - \log_{10}(x-1)^4)$ *1pt.*

$f'(x) = \frac{1}{3}\left(\frac{2x}{x^2 \ln 10} - \frac{4(x-1)^3}{(x-1)^4 \ln 10}\right) = \frac{1}{3 \ln 10}\left(\frac{2}{x} - \frac{4}{(x-1)}\right)$ *1pt.*

$f'(2) = \frac{1}{3 \ln 10}\left(\frac{2}{2} - \frac{4}{(2-1)}\right) = \frac{1}{3 \ln 10}(1-4) = -\frac{1}{\ln 10}$ *1pt.*

3) If $f(x) = (8x-6)^x$, then find $f'(1)$

Let $y = (8x-6)^x$

$\ln y = x \ln(8x-6)$ *1pt.*

$\frac{y'}{y} = x\left(\frac{8}{8x-6}\right) + \ln(8x-6)$ *1pt.*

$y' = \left[x\left(\frac{8}{8x-6}\right) + \ln(8x-6)\right](8x-6)^x$

$f'(1) = \left[\left(\frac{8}{8-6}\right) + \ln(8-6)\right](8-6) = 8 + 2\ln 2$ *1pt.*

4) Find $\lim_{\theta \to \infty} \theta \sin \frac{1}{3\theta}$

Let $x = \frac{1}{3\theta} \quad \to \theta = \frac{1}{3x} \qquad as \quad \theta \to \infty \quad x \to 0$ *1pt.*

$\lim_{\theta \to \infty} \theta \sin \frac{1}{3\theta}$

$= \lim_{x \to 0} \frac{1}{3x} \sin x$ *1pt.*

$= \frac{1}{3} \lim_{x \to 0} \frac{\sin x}{x}$ *1pt.*

$= \frac{1}{3}$ *1pt.*

5) Find the equation of the tangent line to the curve $y = x^2 - 4x + 19$ passes through $P(3,0)$.

Solution:
Since the point $(3,0)$ not on the curve, let the point of contact be when $x = a$, the cprresponding $y - value$ is

$$y = a^2 - 4a + 19$$

So the tangent passes through $(a, a^2 - 4a + 19\,)$ *1pt.*

The slope of the tangent line at the first point is $m = \frac{dy}{dx}$

$\frac{dy}{dx} = 2x - 4 \qquad \rightarrow \quad m = \left.\frac{dy}{dx}\right|_{x=a} = 2a - 4$ *1pt.*

The equation of the tangent line is
$y - (a^2 - 4a + 19) = (2a - 4)(x - a)$ *1pt.*

But since the tangent has to pass through $(3,0)$we must have

$0 - (a^2 - 4a + 19) = (2a - 4)(3 - a)$

$-(a^2 - 4a + 19) = 6a - 2a^2 - 12 + 4a$

$-a^2 + 4a - 19 - 6a + 2a^2 + 12 - 4a = 0$

$a^2 - 6a - 7 = 0 \quad \rightarrow$ *1pt.*

$(a + 1)(a - 7) = 0$

$\rightarrow \quad a = 7 \text{ and } a = -1$ *1pt.*

When $a = 7$
The slope
$$m = \frac{dy}{dx} = 2a - 4 = 10$$ *1pt.*

The equation of the line that passes through *P* is given by
$$y = 10x - 30$$ *1pt.*

When $a = -1$
The slope
$$m = \frac{dy}{dx} = 2a - 4 = -6$$ *1pt.*

The equation of the line that passes through *P* is given by
$$y = -6x + 18$$ *1pt.*