Math 101-151-Class Test 1

Name: ID Serial:

Show all your work. No credits for answers not supported by work

Q4: find the limit if it exists

1. $\lim_{x\to 1} \frac{\sqrt{x^2+1}-\sqrt{2}}{x-1} =$

$\lim_{x\to 1} \frac{\sqrt{x^2+1}-\sqrt{2}}{x-1} = \lim_{x\to 1} \left(\frac{\sqrt{x^2+1}-\sqrt{2}}{x-1}\right)\left(\frac{\sqrt{x^2+1}+\sqrt{2}}{\sqrt{x^2+1}+\sqrt{2}}\right)$ 1pt.

$= \lim_{x\to 1} \frac{x^2+1-2}{(x-1)\sqrt{x^2+1}+\sqrt{2}} = \lim_{x\to 1} \frac{(x-1)(x+1)}{(x-1)\sqrt{x^2+1}+\sqrt{2}}$ 1pt.

$= \lim_{x\to 1} \frac{(x+1)}{\sqrt{x^2+1}+\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$ 2pts.

2. $\lim_{x\to -1} \frac{x+1}{(2x^2+7x+5)^2} =$

$\lim_{x\to -1} \frac{x+1}{(2x^2+7x+5)^2} = \lim_{x\to -1} \frac{x+1}{\left((2x+5)(x+1)\right)^2}$ 1pt.

$= \lim_{x\to -1} \frac{1}{(x+1)(2x+5)^2} = DNE$ 1pt.

Because $\lim_{x\to -1^-} \frac{1}{(x+1)(2x+5)^2} = -\infty \quad and \quad \lim_{x\to -1^+} \frac{1}{(x+1)(2x+5)^2} = +\infty$ 2pts.

3. $\lim_{x\to -\infty} \frac{5x^5+1}{|x|^5-4} =$

$\lim_{x\to -\infty} \frac{5x^5+1}{|x|^5-4} = \lim_{x\to -\infty} \frac{5x^5+1}{-x^5-4}$ 1pt.

$= \lim_{x\to -\infty} \frac{5+\frac{1}{x^5}}{-1-\frac{4}{x^5}} = \frac{5-0}{-1-0} = -5$ 1pt.

Q2: Prove that $\lim_{x \to -1} (2 - 3x) = 5$ using the $\varepsilon - \delta$ definition

Solution

Let Given $\in > 0$ we need to find $\delta > 0$ such that if

$|f(x) - 5| < \in \quad whenever \quad 0 < |x - (-1)| < \delta$

$|2 - 3x - 5| < \in \quad whenever \quad 0 < |x + 1| < \delta$ 1pt.

$|-3 - 3x| < \in \quad whenever \quad 0 < |x + 1| < \delta$ 1pt.

$|(-3)(x + 1)| < \in \quad whenever \quad 0 < |x + 1| < \delta$

$3|x + 1| < \in \quad whenever \quad 0 < |x + 1| < \delta$ 1pt.

$|(x + 1)| < \frac{\in}{3} \quad whenever \quad 0 < |x + 1| < \delta$ 1pt.

If we choose $\delta = \frac{\in}{3}$ then $0 < |x + 1| < \delta = \frac{\in}{3}$ that is $|f(x) - 5| < \in$ 1pt.

Q3: Let $f(x) = x^2 - 4x$ be a function defined over the interval $[-1, 2]$, **use the limit** to find the equation of the tangent line at $P(1, -3)$.

To find the slope of the tangent line

$$m = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{(a+h)^2-4(a+h)-a^2+4a}{h}$$ 1pt.

$$= \lim_{h\to 0} \frac{a^2+2ah+h^2-4a-4h-a^2+4a}{h}$$

$$= \lim_{h\to 0} \frac{2ah+h^2-4h}{h} = \lim_{h\to 0} \frac{h(2a+h-4)}{h}$$ 1pt.

$$= \lim_{h\to 0} 2a + h - 4 = 2a - 4$$ 1pt.

The slope of the tangent line at $a = 1$ is equal to $m = -2$ 1pt.

Thus the equation of the tangent line $y + 3 = -2(x - 1)$ 1pt.

Q4: Find the horizontal and the vertical asymptotes of $f(x) = \frac{1+4e^x}{1-2e^x}$

Solution:

1. To find the H.A, we must evaluate the limits at infinity

$$\lim_{x\to\infty} \frac{1+4e^x}{1-2e^x} = \lim_{x\to\infty} \frac{\frac{1}{e^x}+4}{\frac{1}{e^x}-2} = \frac{0+4}{0-2} = -2$$ 1pt.

and

$$\lim_{x\to-\infty} \frac{1+2e^x}{1-e^x} = \frac{1+0}{1-0} = 1$$ 1pt.

$y = 1 \ \ \& \ \ y = -2 \ \ are \ \ H.A$ ½ pt.

2. To find the V.A, we need to find the limit when the denominator equal to 0

$1 - 2e^x = 0$ 1pt. $\rightarrow$ $e^x = \frac{1}{2}$ $\rightarrow$ $x = \ln\frac{1}{2}$ 1pt.

$x = \ln\frac{1}{2} \quad V.A$ ½ pt.