

Dept of Mathematics and Statistics  
King Fahd University of Petroleum & Minerals

AS482: Actuarial Contingencies II  
Dr. Mohammad H. Omar  
Major 2 Exam Term 151 FORM A  
Sunday Nov 8 2015  
6.00pm-7.20pm

Name \_\_\_\_\_ ID#: \_\_\_\_\_ Serial #: \_\_\_\_\_

**Instructions.**

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 80 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	5		
2	$3+4+5=12$		
3	$4+5=9$		
4	$5+3=8$		
5	$2+4+5=11$		
6	$1+4=5$		
Total	50		

Extra blank page

1. (3+2=5 points) Using the double decrement table below,

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$x$	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(\tau)}$	$p_x^{(\tau)}$	$\ell_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
35	0.011	0.100	0.111	0.889	1000.00	11.00	100.00
36	0.012	0.100	0.112	0.888	889.00	10.27	88.90
37	0.013	0.100	0.113	0.887	789.43	10.26	78.94
38	0.014	0.100	0.114	0.886	700.23	9.80	70.02
39	0.015	0.100	0.115	0.885	620.40	9.31	62.04
40	0.016	0.100	0.116	0.884	549.05	8.78	54.91

find

(a)  $\Pr(K_x^* = 3 \cap J_x = 1)$  for a person age 36.

(b)  $\Pr(K_x^* = 4)$  for a person age 36.

**Solution:** See *Chapter 13 Ex 13.3 pg 316*.

2. (3+4+5=12 points) If  $\mu_{x+t}^{(1)} = 0.10$  and  $\mu_{x+t}^{(2)} = 0.20$  for all  $t$ , find the following:

(a)  ${}_t p_x^{(\tau)}$

(b)  ${}_t q_x^{(1)}$

(c)  ${}_t q_x^{(1)}$

3. (4+5=9 points) If  $q_x^{(1)} = 0.18$  and  $q_x^{(2)} = 0.12$ , and

(a) assuming both decrements are uniformly distributed over the interval  $(x, x + 1]$  in the **multiple decrement** context, find  $q_x'^{(2)}$ .

(b) assuming both decrements are uniformly distributed in their **associated single decrement** tables, find  $q_x'^{(2)}$ .

4. (5+3=8 points) A whole life contract of face amount 80000 is issued to (30). The 20<sup>th</sup> year cash value is 95% of the Net Level Premium (NLP) reserve. The insured has previously borrowed 3500 against the policy. Using the life table in Appendix A with 6% interest,
- Find the *cash value* payable for surrender of the contract at the end of its 20th year.
  - If the policyholder elects the *reduced paid-up* insurance option at the time of surrender instead of taking the cash value, how much reduced paid up insurance could be purchased?

5. (2+4+5=11 points) A person is currently employed at time 0, which we call State 1. Let State 2 denote unemployment and State 3 denote deceased. The transition forces between states are as follows:

(i)  $\lambda_{12}(s) = 0.20 + 0.0002s^2$

(ii)  $\lambda_{13}(s) = \lambda_{23}(s) = 0.05$

(iii)  $\lambda_{21}(s) = 0.80 - 0.04s$

(iv)  $\lambda_{31}(s) = \lambda_{32}(s) = 0$

(a) Draw the transition state diagram for this model.

(b) Find the Kolmogorov differential equations for  ${}_r p_{11}^{(0)}$  and  ${}_r p_{12}^{(0)}$ .

(c) Using one-year time-steps ( $\Delta r = 1$ ) to approximate the solutions to the Kolmogorov differential equation, estimate  ${}_r p_{11}^{(0)}$  and  ${}_r p_{12}^{(0)}$  for  $r = 1, 2, \dots, 20$ . by providing the the missing cell values in the table below:

$r$	${}_r p_{11}^{(0)}$	${}_r p_{12}^{(0)}$	$r$	${}_r p_{11}^{(0)}$	${}_r p_{12}^{(0)}$
1		0.200	11	0.374	0.195
2	0.714	0.188	12	0.341	0.199
3	0.671	0.187	13		
4	0.629	0.186	14	0.279	0.209
5	0.588	0.185	15	0.248	0.215
6	0.550	0.186	16	0.218	0.222
7	0.512	0.186	17	0.188	0.230
8	0.476	0.188	18	0.158	0.240
9	0.441		19		0.250
10	0.407	0.192	20	0.096	0.262

6. (1+4=5 points) For a fully discrete insurance of 1000 on (x), you are given:

- (i)  ${}_4AS = 396.63$  is the *asset share* at the end of year 4.
- (ii)  ${}_5AS = 694.50$  is the *asset share* at the end of year 5.
- (iii)  $G = 281.77$  is the *gross premium*.
- (iv)  ${}_5CV = 572.12$  is the *cash value* at the end of year 5.
- (v)  $c_4 = 0.05$  is the *fraction* of the *gross premium* paid at time 4 for expenses.
- (vi)  $e_4 = 7.0$  is the amount of per policy *expenses* paid at time 4.
- (vii)  $q_{x+4}^{(1)} = 0.09$  is the probability of decrement by *death*.
- (viii)  $q_{x+4}^{(2)} = 0.26$  is the probability of decrement by *withdrawal*.

Calculate  $i$ .

- (a) 0.050
- (b) 0.055
- (c) 0.060
- (d) 0.065
- (e) 0.070

Work Shown (4 points):

Hence the answer is (\_\_\_)

END OF TEST PAPER