Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS482: Actuarial Contingencies II
Dr. Mohammad H. Omar
Major 1 Exam Term 151 FORM A
Sunday Oct 11 2015
6.00pm-7.20pm

Name_____ ID#:_ _ _ _ _ _ _ _ _ Serial #:_ _ _

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 80 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	6		
2	6		
3	1+2+3+3+1=10		
4	6		
5	6		
6	1+5=6		
Total	40		

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1. (6 points) For a 2-state homogeneous Markov survival model, with constant force of transition function $\lambda_{12}(s) = \lambda_1(s) = \lambda$ and $\lambda_{21}(s) = 0$, **solve** the Kolmogorov's differential equation for ${}_np_{12}^{(t)}$.

   Provide the **meaning** of ${}_np_{12}^{(t)}$ and then translate this into **standard actuarial notation**.

2. (6 points) The APV for a *last survivor* whole life insurance on $(\overline{xy})$, with **unit** benefit paid at the **instant of failure** of the status, was calculated assuming **independent** future life times for (x) and (y) with constant hazard rate of 0.06 for each. It is now discovered that although the total hazard rate of 0.06 is correct, the two lifetimes are *not independent* since each includes a **common shock** hazard factor with **constant force** 0.02. The force of interest used in the calculation is $\delta = 0.05$. Calculate the **increase** in the **APV** that results from recognition of the common shock element.

3. (1+2+3+3+1=10 points) .Consider the following probability matrix $\mathbf{P} = \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0 \\ 0.3 & 0.5 & 0 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0.6 & 0.4 \end{bmatrix}$

Answer the following questions:

(a) What type of Markov Chain **model** is represented by this transition matrix?

(b) Draw a diagram that shows the way in which the states communicate.

(c) Assume that the model is *homogeneous*. Provide the 2-step transition matrix P^2.

(d) If the model is *not homogeneous* where the next transition matrix is given by $\begin{bmatrix} 0.8 & 0.1 & 0 & 0.1 \\ 0.3 & 0.5 & 0.1 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0.7 & 0.3 \end{bmatrix}$,

provide the correct value for ${}_2P_{14}^{(0)}$.

(e) If the 4 states of a microwave are 1 = Normal operation, 2 =Minor repair needed , 3 = Total failure, and 4 = Major repair needed, provide the meaning of ${}_2P_{14}^{(0)}$

4. (6 points) Coffee Maker A follows a survival model defined by $\mu_x^A = \dfrac{1.90}{9-x}$, for $0 < x < 9$ and Coffee Maker B follows a survival model defined by $\mu_x^B = \dfrac{1.50}{9-x}$, for $0 < x < 9$. The two Coffee makers have **independent** lifetimes. Find the probability that Coffee maker I **fails before** Coffee Maker B.

5. (6 points) Two washing machine brands, Type A and Type B, follow survival models defined by $\mu_x^A = \ln 1.25$, for $x > 0$, and $\mu_x^B = \dfrac{1}{9-x}$, for $0 < x < 9$, respectively. Given that both brands are currently three years old, and that they have independent lifetimes, find the probability that the **first failure** will occur between ages **4 and 6**.

6. (1+5=6 points) For a select-and-ultimate table with a 2-year select period:

x	$p_{[x]}$	$p_{[x]+1}$	p_{x+2}	$x+2$
48	0.9865	0.9841	0.9713	50
49	0.9858	0.9831	0.9698	51
50	0.9849	0.9819	0.9682	52
51	0.9838	0.9803	0.9664	53

Keith and Clive are independent lives, both age 50. Keith was selected at age 45 and Clive was selected at age 50.

Calculate the probability that **exactly one** will be **alive** at the **end of three** years.

(A) Less than 0.115
(B) At least 0.115, but less than 0.125
(C) At least 0.125, but less than 0.135
(D) At least 0.135, but less than 0.145
(E) At least 0.145

Work Shown (5 points):

Hence the answer is (___)

END OF TEST PAPER