Department of Mathematics and Statistics KFUPM
STAT 319-02 Quiz#6, Time: 50 mins

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A rocket motor is manufactured by bonding together two types of propellants, an igniter and a sustainer. The shear strength of the bond Y (measured in psi) is thought to be a linear function of the age of the propellant X (measured in weeks) when the motor is cast. Twenty observations gave the following summary quantities.

$n = 20,\ \sum y_i = 42648.15,\ \sum x_i = 266.75,\ \sum y_i^2 = 92642656,\ \sum x_i^2 = 4672.44,\ \sum x_i y_i = 527619.9$

a) Calculate S_{XX}, S_{YY} and S_{XY}

b) Estimate the degree of **linear correlation** between and the shear strength of the bond and the age of the propellant. **Interpret** this quantity

Interpretation: ____

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c) Find the estimated regression line. What are your assumptions?

Assumptions:

d) What change in mean shear strength of the bond would be expected for a 1 week change in the age of the propellant?

e) Estimate the error variance.

f) At 5% level of significance, **test** that the higher the age of the propellant, the larger the shear strength of the bond.

g) Calculate the coefficient of determination and interpret it.

Interpretation:____

h) Estimate the mean shear strength of the bond when the age of the propellant is 20 weeks, using 95% confidence level.

$$S_{XX} = \sum x^2 - \frac{1}{n}(\sum x)^2, \quad S_{YY} = \sum y^2 - \frac{1}{n}(\sum y)^2, \quad S_{XY} = \sum xy - \frac{1}{n}(\sum y)(\sum x)$$

$$Y = \beta_0 + \beta_1 X + \epsilon, \quad \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X, \quad e = |Y - \hat{Y}|, \quad \hat{\beta}_1 = \frac{s_{xy}}{s_{xx}}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$SST = SSR + SSE, \quad SSR = \hat{\beta}_1 s_{xy}, \quad SST = s_{yy}, \quad SSE = SST - SSR, \quad \hat{\sigma}^2 = \frac{SSE}{n-2}$$

$$r = \frac{s_{xy}}{\sqrt{s_{xx}s_{yy}}} = \hat{\beta}_1 \sqrt{\frac{s_{xx}}{s_{yy}}} \quad \text{and} \quad R^2 = \frac{\text{Explained Variation}}{\text{Total Variation}} = \frac{SSR}{SST}$$

$\hat{\beta}_0 \pm t_{\frac{\alpha}{2},n-2}\sqrt{\hat{\sigma}^2\left[\frac{1}{n} + \frac{\bar{x}^2}{s_{xx}}\right]}$	$T = \frac{\hat{\beta}_0 - \beta_0}{\sqrt{\hat{\sigma}^2\left[\frac{1}{n} + \frac{\bar{x}^2}{s_{xx}}\right]}}$
$\hat{\beta}_1 \pm t_{\frac{\alpha}{2},n-2}\sqrt{\frac{\hat{\sigma}^2}{s_{xx}}}$	$T = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\frac{\hat{\sigma}^2}{s_{xx}}}}$
$\hat{y}_0 \pm t_{\frac{\alpha}{2},n-2}\sqrt{\hat{\sigma}^2\left[1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{s_{xx}}\right]}$	