King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 301
FINAL EXAM (MASTER)
2014-2015 (143)

Thursday, August 13, 2015 **Allowed Time: 3 Hours**

Name: ____

ID Number: ______________________ **Serial Number:** ______________

Section Number: ____________ **Instructor's Name:** ________________

Instructions:
1. Write neatly and legibly. You may lose points for messy work.
2. **Show all your work.** No points for answers without justification.
3. **Calculators and Mobiles are not allowed**.
4. Make sure that you have 8 different problems

Problem No.	Points	Maximum Points
1		**8**
2		**15**
3		**15**
4		**10**
5		**12**
6		**12**
7		**12**
8		**16**
Total:		**100**

Q1) (8 points). Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 0 & x < 0 \\ e^{-x} & x > 0 \end{cases}$$

Q2) (15 points). The steady-state temperature in a semi-infinite plate is determined from

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \quad y > 0$$

$$u(0, y) = e^{-y}, \quad u(\pi, y) = 0 \ , \quad y > 0$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, \quad 0 < x < \pi,$$

Use the Fourier Transform to solve for $u(x, y)$.

Q3) (15 points). A uniform semi-infinite elastic beam moving along the x -axis with a constant velocity $-v_0$ is brought to a stop by hitting a wall at time t = 0 . The longitudinal displacement $u(x,t)$ is determined from

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad x > 0, \quad t > 0$$

$$u(0,t) = 0, \qquad \lim_{x\to\infty} \frac{\partial u}{\partial x} = 0, \quad t > 0$$

$$u(x,0) = 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = -v_0, \quad x > 0$$

Use Laplace Transform to solve for $u(x,t)$.

Q4) (10 points). Find the steady-state temperature $u(r,\theta)$. in the sphere

$$\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}+\frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta}=0, \quad 0<r<c\,, \quad 0<\theta<\pi$$

$$u(c,\theta)=\cos\theta, \qquad 0<\theta<\pi$$

Q5) (12 points). Let C be the triangle with vertices $(0,0), (0,1), (-1,1)$, then

$$\oint_C e^{x^2} dx + 2\tan^{-1}x\, dy =$$

(a) $\frac{\pi}{2} - \ln 2$

(b) $\frac{\pi}{2} + \ln 2$

(c) $\pi - \ln 4$

(d) $\pi + \ln 4$

(e) 0

Q6) (12 points). The Fourier Bessel series of the function $f(x) = 1,\ 0 < x < 2$, using Bessel functions of order zero that satisfy the boundary condition

$J_0(2\alpha) + 2\alpha J_0'(2\alpha) = 0$ is given by $f(x) = \sum_{i=1}^{\infty} c_i\, J_0(\alpha_i x)$. The value of c_i is

(a) $\frac{4\alpha_i\, J_1(2\alpha_i)}{(4\alpha_i^2+1)J_0^2(2\alpha_i)}$

(b) $\frac{4\alpha_i\, J_1(2\alpha_i)}{J_0^2(2\alpha_i)}$

(c) $\frac{4\alpha_i\, J_1(2\alpha_i)}{(4\alpha_i^2+1)J_0\ (2\alpha_i)}$

(d) $\frac{4\alpha_i\, J_2(2\alpha_i)}{(4\alpha_i^2+1)J_0^2(2\alpha_i)}$

(e) $\frac{4\alpha_i\, J_1(2\alpha_i)}{J_0\ (2\alpha_i)}$

Q7) (12 points). The IBVP

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \qquad 0 < x < \pi,\ t > 0$$

$$u(0,t) = 0, \quad u(\pi,t) = 0, \qquad t > 0$$

$$u(x,0) = \frac{1}{4} x(\pi - x), \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \qquad 0 < x < \pi.$$

has a series solution $u(x,t) = \sum_{n=1}^{\infty} \left(A_n \cos ant + B_n \sin ant\right) \sin nx$. The sum of the coefficients $A_n + B_n =$

(a) $\frac{1}{n^3 \pi} (1 - (-1)^n)$

(b) $\frac{a}{n^3 \pi^3} (1 - (-1)^n)$

(c) $\frac{1}{n^3 \pi^2} (1 + (-1)^n)$

(d) $\frac{a}{n^3 \pi^3} (1 + (-1)^n)$

(e) $\frac{a\, \pi^2}{n^3} (1 + (-1)^n)$

Q8) (16 points). Fill in the blanks:

1) $\mathcal{L}\{\sqrt{t}\, e^{-7t}\} =$ ....

2) $\mathcal{L}\{u(t-\pi)\sin 2t\} =$ ....

3) $\mathcal{L}\{t^2 \cosh 3t\} =$ ....

4) $\mathcal{L}\{(t-1)u(t-1)\} =$ ....

5) $\mathcal{L}^{-1}\left\{\dfrac{1}{3s-1}\right\} =$ ....

6) $\mathcal{L}^{-1}\left\{\dfrac{s+6}{s^2-5}\right\} =$ ....

7) $\mathcal{L}^{-1}\left\{\dfrac{e^{-s}}{s^2+\pi^2}\right\} =$ ....

8) $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-5)^3}\right\} =$ ....