King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 301
EXAM III
2014-2015 (143)

Monday, August 2, 2015 **Allowed Time: 2 Hours**

Name: ____

ID Number: ____________________ **Serial Number:** _______________

Section Number: _____________ **Instructor's Name:** _________________

Instructions:

1. Write neatly and legibly. You may lose points for messy work.

2. **Show all your work.** No points for answers without justification.

3. **Calculators and Mobiles are not allowed**.

4. Make sure that you have 8 different problems

Problem No.	Points	Maximum Points
1		10
2		21
3		17
4		6
5		10
6		13
7		15
8		8
Total:		100

Q1) Classify the given partial differential equation as hyperbolic, parabolic, or elliptic

(a) $\dfrac{\partial^2 u}{\partial x^2} - 3\dfrac{\partial^2 u}{\partial x \partial y} - 2\dfrac{\partial^2 u}{\partial y^2} = 0$

(b) $a^2 \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t}$

Q2) Solve the heat equation $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t} + u\,, \quad 0 < x < \pi\,, \quad t > 0$

subject to the given conditions $u(0,t) = 0\,, \quad u(\pi,t) = 0\,, \quad t > 0$

$u(x,0) = 100\,, \quad 0 < x < \pi$

Q3) Find the Fourier series expansion of the function $f(x)=\begin{cases} 0 & -\pi<x<0 \\ x^2 & 0\le x<\pi \end{cases}$

What does the series converge to when $x=0$?

Q4) Expand $f(x) = x, \ -\pi < x < \pi$ in a sine series.

Q5) Write out the first three nonzero terms in the Fourier-Legendre expansion of the function

$$f(x) = \begin{cases} 0, & -1 < x < 0 \\ 2x, & 0 \le x < 1 \end{cases}$$

Q6) Consider the Parametric Bessel Equation:

$$x^2 y'' + x\, y' + \left(\alpha^2 x^2 - 4\right) y = 0$$

Subject to: y is bounded on $\left[0,5\right], y(5) = 0$

a) State the eigenvalues and the eigenfunctions.

b) Put the differential equation in self-adjoint form

c) Give the orthogonality relation.

Q7) a) show that the set of functions

$$\{\sin nx\}, n = 1, 2, 3, \cdots$$

is orthogonal on the interval $[0, \pi]$.

b) Find the norm of each function in the set.

b) Use the orthogonal set given in part (a) to construct an orthonormal set.

Q8) Expand $f(x) = x^2$, $0 < x < 1$, in a Fourier-Bessel series, using Bessel functions of order two that satisfy the boundary condition $J_2(\alpha) = 0$.