Question 1 a) Verify that $y = C_1 e^x \cos x + C_2 e^x \sin x$ form a general solution of the DE: $y'' - 2y' + 2y = 0$ on $(-\infty, \infty)$. (Do not solve it)

b) Determine the form of a particular solution of the DE:

$$y'' - 2y' + 2y = 5x^2 e^x (\sin x) - 3x e^x (\cos x) + x^2 + 2 \cos x.$$

Question 2 a) Show that the system below has a unique solution:

$$\begin{cases} -2x + 2y - z + 2t &= 0 \\ 2x + y + 2z - 3t &= 0 \\ x - y - z + t &= 1 \\ 3x - 3y + z - t &= 0 \end{cases}$$

b) Solve the above system for t only.

Question 3 Find the inverse of the following matrix:

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 0 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

Question 4 Solve the DE:

$$y^{(4)} + y''' + y' + y = 0 \, .$$

Question 5 Let A and B be $n \times n$ matrices such that $|A| = 8$ and $|B| = 4$. Find $|A^T B^{-1} A^{-2} B^2|$.

Question 6 Determine if the functions below are linearly dependent or linearly independent on $(-\infty, \infty)$:

$$f_1(x) = 3(\sqrt{x} + 1), \ \ f_2(x) = x^2 + 1, \ \ f_3(x) = 6x - 3, \ \ f_4(x) = \sqrt{x} + 2x \,.$$

Question 7 a) Determine whether $W_1 = \{(x, y) \in \mathbb{R}^2 \text{ with } |x| = |y|\}$ is a subspace of $\mathbb{R}^2$.

b) Determine whether $W_2 = \{A \in M_{2,2} \text{ with } |A| = 0\}$ is a subspace of $M_{2,2}$ where $M_{2,2}$ is the set of all 2×2 real matrices.

Question 8 a) Let $W = \{X \in \mathbb{R}^6 \text{ such that } BX = \mathbf{0}\}$ where

$$B = \begin{pmatrix} 1 & -1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 0 & 0 & 5 \end{pmatrix}.$$

Find a basis of W and then, determine its dimension.

b) Let V be the space of all continuous functions f defined on the interval $[-1, 1]$ such that f is a polynomial of degree ≤ 1 on $[-1, 0]$ and a polynomial of degree ≤ 2 on $[0, 1]$. Find the dimension of V.