King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 201 Major Exam II
The Third Semester of 2014-2015 (143)
Time Allowed: 120 Minutes

Name: _____ ID#: _____

Section/Instructor: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.

Question #	Marks	Maximum Marks
1		12
2		10
3		14
4		10
5		14
6		16
7		10
8		14
Total		100

Q:1 (12 points) Find the distance from the point $A(1,-1,-2)$ to the plane containing the points $P(1,3,2), Q(3,-1,6)$, and $R(5,2,0)$.

Q:2 (10 points) Sketch the region between the surfaces

$$x = \sqrt{y^2 + z^2} \quad \text{and} \quad y^2 + z^2 = 1, \;\; 2 \leq x \leq 3$$

Q:3 (14 points) Let $f(x,y) = \sqrt{x^2+y^2-1} \quad + \quad \ln(4-x^2-y^2)$.

(i) Find and sketch the domain of f.

(ii) Find the range of f.

(iii) Find an equation for the **level curve** through the point $(1,1)$.

Q:4 (10 points) Let $f(x,y) = \dfrac{x^3 y}{x^6 + y^2}$.

(i) Evaluate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along the line $x = my$, where m is a constant.

(ii) Evaluate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along the curve $y = x^3$.

(iii) Does the limit $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exist ?

Q:5 a (4 points) Identify the quadratic surface $x^2 - 2x + y + z^2 - 4z + 5 = 0$

(b) (5 points) Find the point of intersection of lines, if they intersect

$$L1 : x = 3 + 2t,\ y = -1 + 4t,\ \ z = 2 - t, \qquad -\infty < t < \infty$$
$$L2 :\ x = 1 + 4s,\ y = 1 + 2s,\ z = -3 + 4s, \qquad -\infty < s < \infty$$

(b) (5 points) Compute $\dfrac{\partial^2 f}{\partial y\ \partial x}$ for $f(x, y) = x^y$ at point $(2, 3)$.

Q:6 (a) (8 points) Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if

$$\tan^{-1}\left(\frac{x}{z}\right) \;=\; \ln(y+z).$$

(b) (8 points) Find $\dfrac{\partial w}{\partial v}$, when $u=-1, v=1$ if $w = xy + \ln z,\;\; x=\dfrac{v^2}{u},\;\; y=u+v,\;\; z=\cos u.$

Q:7 (10 points) (a) Find the directional derivative of $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right) + \sqrt{3}\sin^{-1}\left(\frac{xy}{2}\right)$ at the point $P(1,1)$ in the direction $\overrightarrow{u} = 3\hat{i} - 4\hat{j}$

Q:8 (14 pts) Find the linearization $L(x, y, z)$ of the function $f(x, y, z) = x^2 + xy + yz + \frac{1}{4}z^2$ at point $P(1, 1, 2)$. Then find an upper bound for the magnitude of the error E in the approximation $f(x, y, z) = L(x, y, z)$ over the region:

$$R : |x - 1| \leq 0.01,\ |y - 1| \leq 0.01,\ |z - 2| \leq 0.08.$$