King Fahd University of Petroleum and Minerals
Department of Mathematics and Statisitics

Math 102
Final Exam
Term 143
Thursday 13/08/2015
Net Time Allowed: 180 minutes

MASTER VERSION

1. $\displaystyle\int \frac{(1+\sqrt{x})^{2/3}}{\sqrt{x}}\,dx =$

$u = 1+\sqrt{x} \implies du = \frac{1}{2\sqrt{x}}\,dx$

$I = 2\int u^{2/3}\,du$

$= 2\cdot\frac{3}{5}u^{5/3} + C$

$= \frac{6}{5}(1+\sqrt{x})^{5/3} + C$

(a) $\frac{6}{5}(1+\sqrt{x})^{5/3}+C$

(b) $\frac{2}{5}\ln(1+\sqrt{x})+C$

(c) $\frac{3}{2}(1+\sqrt{x})^{4/3}+C$

(d) $\frac{2}{3}\ln(1+\sqrt{x})-\ln\sqrt{x}+C$

(e) $\dfrac{2}{3(1+\sqrt{x})^{1/3}}+C$

2. $\displaystyle\int_{\pi/2}^{\pi}\sqrt{1-\sin^2 t}\,dt = \int_{\pi/2}^{\pi}\sqrt{\cos^2 t}\,dt = \int_{\pi/2}^{\pi}|\cos t|\,dt$

$= \int_{\pi/2}^{\pi} -\cos t\,dt = -\sin t\Big]_{\pi/2}^{\pi}$

$= -(\sin\frac{\pi}{4} - \sin\frac{\pi}{2})$

$= -(0-1)$

$= 1$

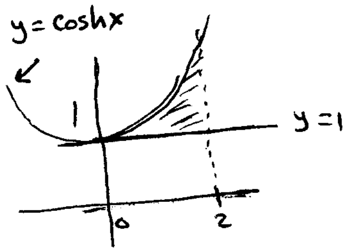
(a) 1

(b) π

(c) $\sqrt{2}$

(d) -2

(e) -1



3. The **area** of the region enclosed by the curves $y = \cosh x$, $y = 1$, and $x = 2$ is equal to

(a) $-2 + \sinh 2$

(b) $\sinh 2$

(c) $\cosh 2$

(d) $2 + \sinh 2$

(e) $-1 + \cosh 2$

$$A = \int_0^2 \cosh x - 1 \, dx$$
$$= \sinh x - x \Big]_0^2$$
$$= (\sinh 2 - 2) - (0 - 0)$$
$$= -2 + \sinh 2$$

4. If $f(x) = \int_1^{2x} \frac{16}{8+t^3}\,dt$, then $f'(x) = \frac{16}{8+(2x)^3} \cdot \frac{d}{dx}[2x]$

$$= \frac{16}{8+8x^3} \cdot 2$$
$$= \frac{2}{1+x^3} \cdot 2$$
$$= \frac{4}{1+x^3}$$

(a) $\dfrac{4}{1+x^3}$

(b) $\dfrac{16}{8+x^3}$

(c) $\dfrac{32}{8+x^3}$

(d) $\dfrac{16}{1+8x^3}$

(e) 0

5. $\int_0^1 x(1-x)^7\,dx =$

$U = 1-x \Rightarrow du = -dx$

$\cdot\ x=0 \Rightarrow u=1$

$\cdot\ x=1 \Rightarrow u=0$

(a) $\frac{1}{72}$

(b) $\frac{1}{63}$

(c) $\frac{1}{8}$

(d) $\frac{1}{9}$

(e) $\frac{1}{7}$

$I = -\int_1^0 (1-u)\,u^7\,du$

$= \int_0^1 u^7 - u^8\,du$

$= \frac{1}{8}u^8 - \frac{1}{9}u^9 \Big]_0^1$

$= \left(\frac{1}{8} - \frac{1}{9}\right) - (0-0)$

$= \frac{1}{72}$

6. $\int (\sec x\,\tan x)^3\,dx =$ $\int \sec^3 x\,\tan^3 x\,dx = \int \sec^2 x \cdot \tan^2 x \cdot \sec x \tan x\,dx$

$= \int \sec^2 x\,(\sec^2 x - 1) \cdot \sec x \tan x\,dx$

$= \int (\sec^4 x - \sec^2 x) \cdot \sec x \tan x\,dx$

$u = \sec x \Rightarrow du = \sec x \tan x\,dx$

$= \int u^4 - u^2\,du$

$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$

$= \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

(a) $\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

(b) $\frac{1}{4}(\sec x\,\tan x)^4 + C$

(c) $\frac{1}{3}(\sec x\,\tan x)^3 + C$

(d) $\frac{1}{4}\sec^4 x - \frac{1}{2}\sec^2 x + C$

(e) $\frac{1}{3}\tan^3 x - \tan x + C$

7. The improper integral $\int_{-\infty}^{0} \frac{x}{(x^2+2)^{3/2}}\,dx$ is

$= \lim_{t \to \infty} \int_t^0 x\,(x^2+2)^{-3/2}\,dx$

$= \lim_{t \to -\infty} \frac{1}{2} \cdot -2\,(x^2+2)^{-1/2} \Big]_t^0$

$= \lim_{t \to \infty} -\left(2^{-1/2} - (t^2+2)^{-1/2}\right)$

$= -\left(\frac{1}{\sqrt{2}} - 0\right)$

$= -\frac{1}{\sqrt{2}}$ Conv.

(a) convergent and its value is $-\frac{1}{\sqrt{2}}$

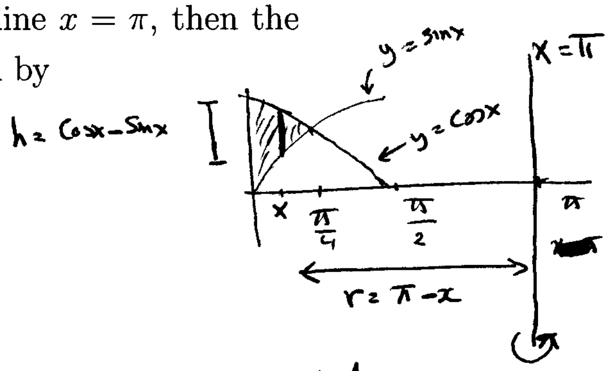
(b) convergent and its value is 1

(c) convergent and its value is $\frac{3}{2}$

(d) convergent and its value is $\frac{\sqrt{2}}{4}$

(e) divergent

8. If the region bounded by the curves $y = \sin x$ and $y = \cos x$, for $0 \le x \le \frac{\pi}{4}$, is revolved about the line $x = \pi$, then the **volume** of the generated solid is given by



(a) $2\pi \int_0^{\pi/4} (\pi - x)(\cos x - \sin x)\,dx$

(b) $2\pi \int_0^{\pi/4} (x - \pi)(\cos x - \sin x)\,dx$

(c) $2\pi \int_0^{\pi/4} (\pi - x)(\sin x - \cos x)\,dx$

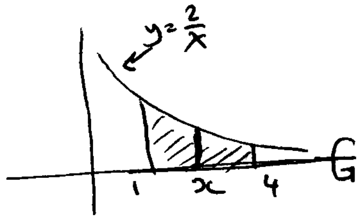
(d) $2\pi \int_0^{\pi/4} (x + \pi)(\sin x - \cos x)\,dx$

(e) $2\pi \int_0^{\pi/4} (\pi + x)(\sin x + \cos x)\,dx$

Shell method

$V = \int_0^{\pi/4} 2\pi \cdot (\pi - x)\,(\cos x - \sin x)\,dx$

9. The **volume** of the solid generated by rotating the region enclosed by the curves $y = \dfrac{2}{x}$, $y = 0, x = 1, x = 4$ about the x-axis is equal to



Disk method

$$V = \pi \int_1^4 \left(\frac{2}{x}\right)^2 dx$$
$$= 4\pi \int_1^4 \frac{1}{x^2}\, dx$$
$$= 4\pi \cdot -\frac{1}{x}\Big]_1^4$$
$$= 4\pi \left(-\frac{1}{4} + 1\right)$$
$$= 4\pi \cdot \frac{3}{4} = 3\pi$$

(a) 3π

(b) π

(c) 10π

(d) 5π

(e) 6π

10. $\displaystyle\int \sqrt{x}\, \ln\sqrt{x}\, dx =$

By Parts $u = \ln\sqrt{x}$ $dv = \sqrt{x}\, dx$

$du = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\, dx$ $v = \frac{2}{3} x^{3/2}$

$$I = \frac{2}{3} x^{3/2} \ln\sqrt{x} - \frac{1}{3} \int x^{1/2}\, dx$$
$$= \frac{2}{3} x^{3/2} \ln\sqrt{x} - \frac{1}{3} \cdot \frac{2}{3} x^{3/2} + C$$
$$= \frac{2}{3} x^{3/2} \left(\ln\sqrt{x} - \frac{1}{3}\right) + C$$

(a) $\dfrac{2}{3} x^{3/2} \left(\ln\sqrt{x} - \dfrac{1}{3}\right) + C$

(b) $\dfrac{1}{3} x^{3/2} \left(\ln\sqrt{x} + \dfrac{2}{3}\right) + C$

(c) $\dfrac{2}{3} x^{3/2} \ln\sqrt{x} - \dfrac{2}{3}\sqrt{x} + C$

(d) $\ln\sqrt{x} - \dfrac{2}{9}\sqrt{x} + C$

(e) $\sqrt{x}\ln\sqrt{x} - \dfrac{2}{9} x^{3/2} + C$

11. The **length** of the curve $y = \ln x,\ 1 \le x \le e$, is given by

(a) $\displaystyle\int_{\pi/4}^{\tan^{-1} e} \frac{\sec^3\theta}{\tan\theta}\,d\theta$

(b) $\displaystyle\int_{\pi/4}^{\tan^{-1} e} \frac{\sec^2\theta}{\tan\theta}\,d\theta$

(c) $\displaystyle\int_{1}^{e} \sqrt{1+\frac{1}{x}}\,dx$

(d) $\displaystyle\int_{1}^{e} \frac{\sqrt{1+x}}{x^2}\,dx$

(e) $\displaystyle\int_{\pi/4}^{\tan^{-1} e} \frac{\sin\theta}{\cos^3\theta}\,\theta$

$$L = \int_1^e \sqrt{1+\frac{1}{x^2}}\,dx$$

$$= \int_1^e \frac{\sqrt{x^2+1}}{x}\,dx$$

$x = \tan\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$= \int_{\pi/4}^{\tan^{-1}e} \frac{\sqrt{\tan^2\theta+1}}{\tan\theta}\cdot \sec^2\theta\,d\theta$$

$$= \int_{\pi/4}^{\tan^{-1}e} \frac{\sec\theta}{\tan\theta}\cdot \sec^2\theta\,d\theta$$

$$= \int_{\pi/4}^{\tan^{-1}e} \frac{\sec^3\theta}{\tan\theta}\,d\theta$$

12. $\displaystyle\int \frac{1}{2x^2-x-1}\,dx =$ $\displaystyle\int \frac{1}{(x-1)(2x+1)}\,dx = \int \frac{A}{x-1} + \frac{B}{2x+1}\,dx$

(a) $\displaystyle\frac{1}{3}\ln\left|\frac{x-1}{2x+1}\right| + C$

(b) $\displaystyle\frac{2}{3}\ln|2x^2-x-1| + C$

(c) $\displaystyle\frac{1}{3}\ln|x-1| - \frac{2}{3}\ln|2x+1| + C$

(d) $\displaystyle\frac{1}{3}\ln|x-1| - \frac{1}{2}\ln|2x+1| + C$

(e) $\displaystyle\frac{4}{3}\ln\left|\frac{x+1}{2x-1}\right| + C$

$1 = A(2x+1) + B(x-1)$

$x=1 \Rightarrow 1 = 3A \Rightarrow A = \frac{1}{3}$

$x=-\frac{1}{2} \Rightarrow 1 = -\frac{3}{2}B \Rightarrow B = -\frac{2}{3}$

$= A\ln|x-1| + B\cdot\frac{1}{2}\ln|2x+1| + C$

$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|2x+1| + C$

$= \frac{1}{3}\ln\left|\frac{x-1}{2x+1}\right| + C$

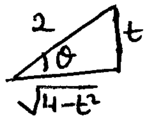
$x = t^2 \Rightarrow dx = 2t\,dt$

13. $\int \sqrt{\frac{4-x}{x}}\,dx =$ **(Hint : Let** $t = \sqrt{x}$**)**

$I = \int \frac{\sqrt{4-t^2}}{t} \cdot 2t\,dt$

$= 2\int \sqrt{4-t^2}\,dt \quad , \quad t = 2\sin\theta,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$= 2\int 2\cos\theta \cdot 2\cos\theta\,d\theta$



$= 8\int \frac{1+\cos(2\theta)}{2}\,d\theta$

$= 4\left[\theta + \frac{1}{2}\sin(2\theta)\right] + C$

$= 4\theta + 4\sin\theta\cos\theta + C$

$= 4\sin^{-1}\left(\frac{t}{2}\right) + 4 \cdot \frac{t}{2}\,\frac{\sqrt{4-t^2}}{2} + C$

$= 4\sin^{-1}\left(\frac{\sqrt{x}}{2}\right) + \sqrt{x}\,\sqrt{4-x} + C$

$= 4\sin^{-1}\left(\frac{\sqrt{x}}{2}\right) + \sqrt{4x-x^2} + C$

(a) $4\sin^{-1}\left(\frac{\sqrt{x}}{2}\right) + \sqrt{4x-x^2} + C$

(b) $2\sin^{-1}(2\sqrt{x}) + \sqrt{x^2-4x} + C$

(c) $4\sin^{-1}\left(\frac{\sqrt{x}}{4}\right) + \sqrt{4x-x^2} + C$

(d) $2\sin^{-1}(\sqrt{x}) + \sqrt{4x-x^2} + C$

(e) $\sin^{-1}(\sqrt{4-x}) + \frac{1}{2}\sqrt{x} + C$

14. $\int 4\cot x\,\csc(2x)\,dx = \int 4 \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\sin(2x)}\,dx$

$= \int 4 \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{2\sin x\cos x}\,dx$

$= \int 2 \cdot \frac{1}{\sin^2 x}\,dx$

$= 2\int \csc^2 x\,dx$

$= -2\cot x + C$

(a) $-2\cot x + C$

(b) $2\cos(2x) + C$

(c) $-2\tan x + C$

(d) $4\sin x + C$

(e) $2\sec x + C$

15. The series $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{2n-1}\right)$ is

$\lim_{n\to\infty} \ln\left(\frac{n+1}{2n-1}\right) = \ln\left(\frac{1}{2}\right) = -\ln 2 \neq 0$

$\Rightarrow$ Series div by n^{th} term test.

(a) divergent by the n^{th} – term test for divergence

(b) convergent by the n^{th} – term test for divergence

(c) convergent by the integral test

(d) convergent by the comparison test

(e) divergent by the integral test

16. Which one of the following series is **Divergent**?

(a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n-1}{n+1}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln(n+1)}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$

(e) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$

- b, c, d, e all Conv. by the Alternating Series Test.
- (a) div. by the n^{th} term test for div.

17. The series $\sum_{n=1}^{\infty} \frac{n^n}{2^{(n^2)}}$ is

Use the Root Test

$\lim_{n\to\infty} \sqrt[n]{\frac{n^n}{2^{(n^2)}}} = \lim_{n\to\infty} \frac{n}{2^n} \overset{LR}{=} \lim_{n\to\infty} \frac{1}{2^n \cdot \ln 2} = 0 < 1$

$\Rightarrow$ Conv.

(a) convergent

(b) divergent

(c) a series for which no test is applicable

(d) divergent by the limit comparison test

(e) a convergent geometric series.

18. The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{e^{-2n}}{2^{n-2}}$ is

$\sum_{n=1}^{\infty} (-1) \cdot 2^2 \cdot \left(\frac{-1}{2e^2}\right)^n$, a geometric Series with $r = \frac{-1}{2e^2}$ & $a = \frac{2}{e^2}$

. Since $|r| = \frac{1}{2e^2} < 1$, then the series Conv. & its Sum is

$\frac{a}{1-r} = \frac{\frac{2}{e^2}}{1+\frac{1}{2e^2}}$

$= \frac{4}{2e^2+1}$

(a) convergent and its sum is $\frac{4}{2e^2+1}$

(b) convergent and its sum is $\frac{1}{e^2+2}$

(c) convergent and its sum is $\frac{e}{e+2}$

(d) convergent and its sum is $\frac{e^2}{e+1}$

(e) divergent

19. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+2}{n^2+\sqrt{n}}$ is

- Series Conv. by Alternating series test (check
- $\sum \left|(-1)^{n+1}\frac{n+2}{n^2+\sqrt{n}}\right| = \sum \frac{n+2}{n^2+\sqrt{n}}$ div. by the Limit Comparison test with $\sum \frac{1}{n}$. (check)

So the given series Conv. Conditionally.

(a) conditionally convergent

(b) absolutely convergent

(c) divergent

(d) convergent by the ratio test

(e) divergent by the integral test.

20. The sum of the series $\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{2\cdot n!}$ is

$$\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{n!}$$

$$\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-\pi^2)^n}{n!}$$

$$\frac{1}{2}e^{-\pi^2} = \frac{1}{2e^{\pi^2}}$$

(a) $\frac{1}{2\,e^{\pi^2}}$

(b) $2\,e^{\sqrt{\pi}}$

(c) $\frac{1}{2}e^{\pi^2}$

(d) $\frac{1}{2}e^{-\pi}$

(e) ∞

21. The interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{2^n}{n} x^{2n}$ is

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{2^{n+1}}{n+1} x^{2n+2} \cdot \frac{n}{2^n x^{2n}} \right|$

$= \lim_{n\to\infty} \frac{2n}{n+1} |x|^2 = 2|x|^2$

Conv. if $2|x|^2 < 1 \Rightarrow |x| < \frac{1}{\sqrt{2}}$

$\Rightarrow -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

Endpts: $x = -\frac{1}{\sqrt{2}} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n}{n} \cdot \frac{1}{2^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ div. (the Harmonic series)

$x = \frac{1}{\sqrt{2}} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n}{n} \cdot \frac{1}{2^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ " "

Interval of Conv. is $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(a) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(c) $(-\infty, \infty)$

(d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(e) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} \Rightarrow L = \infty$ or $L > 1$

22. Let $a_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ is divergent by using the **ratio test**, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$

$\Rightarrow \lim_{n\to\infty} \frac{1/a_{n+1}}{1/a_n} = \lim_{n\to\infty} \frac{a_n}{a_{n+1}} = \frac{1}{L} < 1$ or $0 < 1$

$\Rightarrow \sum \frac{1}{a_n}$ Conv. by Ratio test.

(a) is convergent by the ratio test

(b) is divergent by the ratio test

(c) is a series for which the ratio test is inconclusive

(d) is divergent by the n^{th} – term test for divergence

(e) is convergent by the integral test.

23. The series $\sum_{n=1}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n}\right)$ is

$$\cdot\ S_n = \sum_{i=1}^{n}\left(\frac{1}{i+2}-\frac{1}{i}\right) = -\sum_{i=1}^{n}\left(\frac{1}{i}-\frac{1}{i+2}\right)$$
$$= -\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+2}\right)\right]$$
$$= -\left[1+\frac{1}{2}-\frac{1}{n+2}\right] = -\frac{3}{2}+\frac{1}{n+2}$$
$$\cdot\ \lim_{n\to\infty} S_n = -\frac{3}{2}$$

Conv. & Sum is $-\frac{3}{2}$

(a) convergent and its sum is $\frac{-3}{2}$

(b) convergent and its sum is $\frac{1}{3}$

(c) convergent and its sum is -1

(d) convergent and its sum is $\frac{1}{2}$

(e) divergent

24. The **Taylor series** of $f(x) = 2^{-x}$ about $a = 1$ is

(a) $\sum_{n=0}^{\infty}(-1)^n\frac{(\ln 2)^n}{2\cdot n!}(x-1)^n$

(b) $\sum_{n=0}^{\infty}\frac{(-2)^n \ln 2}{n!}(x-1)^n$

(c) $\sum_{n=0}^{\infty}\frac{(-2)^n \ln 2}{n!}(x+1)^n$

(d) $\sum_{n=0}^{\infty}\frac{2^{-n} \ln 2}{n}(x+1)^n$

(e) $\sum_{n=0}^{\infty}\frac{2(-1)^n(\ln 2)^n}{n}(x-1)^n$

$$f'(x) = 2^{-x}(-1)\ln 2$$
$$f''(x) = 2^{-x}(-\ln 2)^2$$
$$f'''(x) = 2^{-x}(-\ln 2)^3$$
$$\vdots$$
$$f^{(n)}(x) = 2^{-x}(-\ln 2)^n$$
$$f^{(n)}(1) = 2^{-1}\cdot(-\ln 2)^n$$

The Taylor Series is

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(1)}{n!}(x-1)^n$$
$$\sum_{n=0}^{\infty}\frac{2^{-1}(-\ln 2)^n}{n!}(x-1)^n$$
$$= \sum_{n=0}^{\infty}(-1)^n\frac{(\ln 2)^n}{2\cdot n!}(x-1)^n$$

25. The coefficient of x^4 in the **binomial series** of $(1+2x^2)^{-1/3}$ is

(a) $\frac{8}{9}$

(b) $\frac{4}{9}$

(c) $-\frac{8}{9}$

(d) $-\frac{16}{9}$

(e) $\frac{7}{6}$

$$\frac{(-\frac{1}{3})(-\frac{1}{3}-1)}{2!}(2x^2)^2$$

$$\frac{-\frac{1}{3}\cdot -\frac{4}{3}}{2}\cdot 4x^4$$

$$\frac{4}{9}\cdot 2x^4 = \frac{8}{9}x^4$$

26. $\int_0^1 x\sin(x^3)\,dx =$

(a) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!\cdot(6n+5)}$

(b) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!\cdot(2n+2)}$

(c) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!\cdot(6n+4)}$

(d) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!\cdot(3n+4)}$

(e) $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!\cdot(4n+3)}$

$$\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}, \quad |x|<\infty$$

$$\sin(x^3) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{6n+3}}{(2n+1)!}, \quad |x^3|<\infty$$

$$x\sin(x^3) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{6n+4}}{(2n+1)!}, \quad |x|<\infty$$

$$\int_0^1 x\sin(x^3)dx = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\int_0^1 x^{6n+4}dx$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\cdot\left.\frac{x^{6n+5}}{6n+5}\right]_0^1$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\cdot\frac{1}{6n+5}$$

27. For some suitable values of x, the **Maclaurin series** for $f(x) = \dfrac{3}{1+9x^3}$ is given by

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad , \ |x|<1$$

sub $-9x^3$

$$\frac{1}{1+9x^3} = \sum_{n=0}^{\infty} (-9x^3)^n \quad , \ |-9x^3|<1$$

$$\frac{1}{1+9x^3} = \sum_{n=0}^{\infty} (-1)^n 9^n x^{3n} \quad , \ |x| < \frac{1}{\sqrt[3]{9}}$$

$$\frac{3}{1+9x^3} = 3\sum_{n=0}^{\infty} (-1)^n \cdot 3^{2n} \cdot x^{3n}$$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot 3^{2n+1} \cdot x^{3n} \quad , \ |x| < \frac{1}{\sqrt[3]{9}}$$

(a) $\sum_{n=0}^{\infty} (-1)^n\, 3^{2n+1} \cdot x^{3n}$

(b) $\sum_{n=0}^{\infty} (-1)^n\, 9^n \cdot x^{3n}$

(c) $\sum_{n=0}^{\infty} (-1)^n\, 9^{n+1} \cdot x^n$

(d) $\sum_{n=0}^{\infty} 3^{2n} \cdot x^{3n+1}$

(e) $\sum_{n=0}^{\infty} 3^{2n+1} \cdot x^{2n+1}$

28. The first three nonzero terms of the **Maclaurin series** for $f(x) = (\sin x)\ln(1+x)$ are

$$\left(x - \frac{x^3}{6} + \frac{x^5}{5!} + \cdots\right)\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\right)$$

$$= x^2 - \frac{x^3}{2} + \frac{x^4}{3} - \frac{x^5}{4} + \cdots$$

$$- \frac{x^4}{6} + \frac{x^5}{12} - \frac{x^6}{18} + \cdots$$

$$+ \cdots$$

$$= x^2 - \frac{x^3}{2} + \left(\frac{1}{3} - \frac{1}{6}\right)x^4 + \cdots$$

$$= x^2 - \frac{1}{2}x^3 + \frac{1}{6}x^4 + \cdots$$

(a) $x^2 - \frac{1}{2}x^3 + \frac{1}{6}x^4 - \ldots$

(b) $x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^5 + \ldots$

(c) $\frac{1}{6}x^4 - \frac{1}{12}x^5 + \frac{1}{48}x^6 - \ldots$

(d) $x^2 + \frac{1}{18}x^4 - \frac{1}{72}x^6 + \ldots$

(e) $x - \frac{1}{6}x^3 + \frac{1}{24}x^5 - \ldots$