Department of Mathematics and Statistics KFUPM **Time: 3 hours**

STAT 591 Final Exam

Name: ----------------------------------- **ID: ------------------------------**

Q1. **(1x5=5 points)** Provide a brief description of the following terms.

1. **Producer's Risk:**

2. **Average Sample Number:**

3. **Limiting Quality:**

4. **Average Outgoing Quality Limit:**

5. **Average Total Inspection:**

Q.2. (5+10=15 points) Let $\underset{\sim}{X}$ be a continuous random vector (of correlated quality characteristics of interest in a process) with mean vector $\underset{\sim}{\mu}$ and variance covariance matrix Σ and is normally distributed, i.e. $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$. Let $\underset{\sim}{x}_1, \underset{\sim}{x}_2, ..., \underset{\sim}{x}_n$ be a random sample of size n from the process. Consider p=2, n=4, and the case of known parameters as specified below:

$$\underset{\sim}{\mu}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \Sigma_0 = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$$

Using this information:

a). Provide the following for Chi-squared control chart:

i). Plotting statistic:

ii). Control limits at the false alarm rate of 0.005:

b). Provide the following for the generalized variance chart based on $|S|^{1/2}$:

i). Control limits (two-sided) at the false alarm rate of 0.005

ii). If the process variance covariance matrix shifts to a new level defined as: $|\Sigma|_1^{1/2} = \delta|\Sigma|_0^{1/2}$. Using the limits computed in part (i), evaluate the power at δ = 0.5, 1.00, 1.5 and find their corresponding ARL1 values.

Q.3. (5 points) For a location control chart, following is the table of ARL values against their respective shifts quantified by δ. Using this information, compute the value of EQL value and interpret it.

δ	**0.00**	**0.20**	**0.40**	**0.60**	**0.80**	**1.00**	**1.20**	**1.40**	**1.60**	**1.80**	**2.00**
ARL	375.38	180.57	57.33	20.97	9.03	4.57	2.70	1.83	1.40	1.18	1.08

Q.4. (4x5=20 points) A control chart indicates that the current process fraction nonconforming is 0.02. If 50 items are inspected each day. Using this information do the following (use exact sampling distribution for computations).

i) Calculate 3-sigma control limits to monitor fraction non-conforming

ii) Compute false alarm rate for the limits obtained in part (i)

iii) If the fraction nonconforming shifts to 0.04, what is the probability of detecting a shift in the fraction nonconforming to 0.04 on the first day after the shift?

iv) If the fraction nonconforming shifts to 0.04, what is the probability of detecting a shift in the fraction nonconforming to 0.04 by the end of the third day after the shift?

v) Repeat parts (ii) and (iii) using an appropriate approximation, and compare the results with the exact results obtained above.

Q.5. (5+1+4=10 points)
i) Derive the formula of Average sample number (ASN) for double sampling plan.

ii) Mention the restrictions if you have imposed any in your derivation in part (i) above.

iii). Discuss the maximum and minimum values of ASN derived in part (i) above and what types of lots are associated with these values?

Q.6. (4+4+2=10 points)

For the lot quality p=0.05 and N=1000, compute Average total inspection (ATI) for the following two plans:

i) Single sampling plan with n=60, c=3

ii) a corresponding double sampling plan with n1= 40, n2=90, c1=2, c2=5.

iii) Compare the results of the above two parts and comment.

Q7 (4x5=20 points).

Consider a random vector X of original variables and a corresponding random vector Y of principal components. In order to monitor the parameters of interest we may use original vector X or the corresponding principal components based vector Y.
(a) Discuss the procedural details of both the approaches;
(b) Provide literature review on both;
(c) Do a comparative analysis of the two approaches and give the findings;
(d) Give your recommendations;
(e) For a practical dataset, apply both the approaches and examine the outcomes. Check whether these outcomes are in accordance with some of the findings reported in part (c) above.

(Q7 is Open Book): Please send me the soft copy of your solution of Q7 before 11:00 am tomorrow (May 27, 2015) on riazm@kfupm.edu.sa