8.1.4. Let $X_1, X_2, \ldots, X_{10}$ be a random sample of size 10 from a normal distribution $N(0, \sigma^2)$. Find a best critical region of size $\alpha = 0.05$ for testing $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 2$. Is this a best critical region of size 0.05 for testing $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 4$? Against $H_1 : \sigma^2 = \sigma_1^2 > 1$?

8.1.5. If $X_1, X_2, \ldots, X_n$ is a random sample from a distribution having pdf of the form $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$, zero elsewhere, show that a best critical region for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$ is $C = \left\{(x_1, x_2, \ldots, x_n) : c \leq \prod_{i=1}^{n} x_i\right\}$.

8.1.6. Let $X_1, X_2, \ldots, X_{10}$ be a random sample from a distribution that is $N(\theta_1, \theta_2)$. Find a best test of the simple hypothesis $H_0 : \theta_1 = \theta_1' = 0$, $\theta_2 = \theta_2' = 1$ against the alternative simple hypothesis $H_1 : \theta_1 = \theta_1'' = 1$, $\theta_2 = \theta_2'' = 4$.

8.1.7. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a normal distribution $N(\theta, 100)$. Show that $C = \left\{(x_1, x_2, \ldots, x_n) : c \leq \overline{x} = \sum_{1}^{n} x_i/n\right\}$ is a best critical region for testing $H_0 : \theta = 75$ against $H_1 : \theta = 78$. Find n and c so that

$$P_{H_0}[(X_1, X_2, \ldots, X_n) \in C] = P_{H_0}(\overline{X} \geq c) = 0.05$$

and

$$P_{H_1}[(X_1, X_2, \ldots, X_n) \in C] = P_{H_1}(\overline{X} \geq c) = 0.90,$$

approximately.

8.1.8. If $X_1, X_2, \ldots, X_n$ is a random sample from a beta distribution with parameters $\alpha = \beta = \theta > 0$, find a best critical region for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$.

8.1.9. Let $X_1, X_2, \ldots, X_n$ be iid with pmf $f(x; p) = p^x(1-p)^{1-x}$, $x = 0, 1$, zero elsewhere. Show that $C = \left\{(x_1, \ldots, x_n) : \sum_{1}^{n} x_i \leq c\right\}$ is a best critical region for testing $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{1}{3}$. Use the Central Limit Theorem to find n and c so that approximately $P_{H_0}\left(\sum_{1}^{n} X_i \leq c\right) = 0.10$ and $P_{H_1}\left(\sum_{1}^{n} X_i \leq c\right) = 0.80$.

8.1.10. Let $X_1, X_2, \ldots, X_{10}$ denote a random sample of size 10 from a Poisson distribution with mean θ. Show that the critical region C defined by $\sum_{1}^{10} x_i \geq 3$ is a best critical region for testing $H_0 : \theta = 0.1$ against $H_1 : \theta = 0.5$. Determine, for this test, the significance level α and the power at $\theta = 0.5$.