**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**
**DEPARTMENT OF MATHEMATICS & STATISTICS**
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**STAT 502: Statistical Inference**
Semester 142
Final Exam (Subjective)
Saturday May 23, 2015
8:00 − 10:00 pm

Q.No.1:- (5+5+5+5=20 points)

(a) If $k$ trials conducted are of Bernoullian type following binomial distribution, find the maximum likelihood estimate of $p$.

(b) Find the maximum likelihood estimate of the parameter $\theta$ of the following distribution,

$$f(x;\theta) = \frac{1}{2}e^{-|x-\theta|}$$

$-\infty < x < \infty$ and $-\infty < \theta < \infty$.

(c) Estimate the parameter $\beta$ of the distribution $f(x;\beta) = \beta e^{-\beta x}$ for $0 \le x \le \infty$, by the method of moments.

(d) Find Bayes estimator of the single parameter $\theta$ of the Poisson distribution $\left(f(x;\theta) = \frac{e^{-\theta}\theta^x}{x!}\right.$ for $x = 0, 1, 2, \dots\dots, n)$ when it is known that the prior distribution of $\theta$ is gamma distribution $\left(g(\theta) = \frac{\beta^\alpha}{\Gamma\alpha}\theta^{\alpha-1}e^{-\beta\theta}\right.$ for $0 \le \theta \le \infty)$.

Q.No.2:- (7 points) Consider a distribution having a pmf of the form $f(x;\theta) = \theta^x(1-\theta)^{1-x}$ for $x = 0, 1$. Let $H_0\colon \theta = \frac{1}{20}$ against $H_1\colon H_0\colon \theta > \frac{1}{20}$. Use the central limit theorem to determine the sample size $n$ of a random sample so that the uniformly most powerful test of $H_0$ against $H_1$ has a power function $\gamma(\theta)$, with approximately $\gamma\left(\frac{1}{20}\right) = 0.05$ and $\gamma\left(\frac{1}{10}\right) = 0.90$.

Q.No.3:- (8 points) Let $X_1, X_2, \dots, X_n$ denote a random sample from a gamma distribution $\left(f(x;\theta) = \frac{\beta^\alpha}{\Gamma\alpha}x^{\alpha-1}e^{-\beta x}\right.$ for $x > 0$, $\mu = \alpha/\beta$ and $\sigma^2 = \alpha/\beta^2)$ with $\alpha = 2$ and $\beta = \theta$. Let $H_0\colon \theta = 1$ against $H_1\colon \theta > 1$. Show that the likelihood ratio test leads to the same critical region as that given by the Neyman-Pearson lemma. Also find the value of $k$ using $\alpha = 0.05$.

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***With the Best Wishes***