KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 142
STAT416 : Stochastic Processes for Actuaries (142)
Third Exam **Monday May 4, 2015**

Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	17	
Two	10	
Three	15	
Four	12	
Five	14	
Six	16	
Seven	16	
Total	100	

Question.1 (**3+2+4+4+4=17-Points**)

Explain the meaning or define the following:

(a) The queueing system $M/M/3$

(b) If $\pi_3 = 0.50$ for an $M/M/5$ queueing system, what is the meaning of this number?

(c) For an $M/M/1$ queueing system, we get $E(L) = 1, E(W_q) = \frac{1}{6}$ what is the meaning of these two numbers?

(d) Define a standard Browanian motion $\{X(t); t \geq 0)\}$

(e) Define a geometric Browanian motion $\{X(t); t \geq 0)\}$

Question.2 **(2+2+3+3=10-Points)** Cars arrive at a gas station according to a Poisson process at rater of 20 cars per hour. The station has only one pumping machine which provides the service (filling the car tank) at rate of one car every two minutes.

(a) Specify the type of queueing system in this station

(b) Find the limiting probability that the station has no cars. (do not derive).

(c) What is the probability that a customer waits more than 5 minutes in this system?

(d) What is the probability that a customer waits less than 3 minutes before filling his car's tank?

Question.3 **(15-Points)** Derive the limiting probabilities for an $M/M/k$ queueing system.

Question.4 **(4+8=12-Points)** Consider the tandem or sequential system with two servers. Assume that the arrival rate at each server is λ and the service rate is μ_1, μ_2 for server 1 and 2 respectively.

(a) Use the following graph to set up the balance equation

(b) Assume that the customers from outside the system arrive at server 1 at a Poisson rate of (6) and at server 2 at a Poisson rate of (4). The service rates of servers 1 and 2 are respectively (15) and (18). A customer upon completion of service at server 1 will go to server 2 twice the chance that he will leave the system ($\pi_{12} = \frac{2}{3}, \pi_{11} = 0$); whereas a departure from server 2 is equally likely to go to server 1 or departs the system otherwise ($\pi_{21} = \frac{1}{2}, \pi_{22} = 0$). Determine the limiting probability of having two customers at server 1 and three customers at server 2.

Question 5. **(6+3+2+3=14-Points)** The stock price of company A follows a standard Browanian motion. Given that the stock price when $t = 5$ is \$70, where t is measured in weeks. Find the following:

(a) What is the probability that the stock price will increase at least \$3 by week 10?

(b) Find $Var(3X(5) - 2X(7))$

(c) Find $Cov(X(12), X(2))$

(d) What is the distribution of $X(6) + X(8)$.

Question 6. (**6+10=16-Points**) Suppose that the stock price of company B changes according to a Browanian motion with $\sigma^2 = 16$, and the time t is measured in months.

(a) What is the probability that the stock price will be less than 9 in three months?

(b) Find $P\{X(5) - 2X(3) \leq 15\}$

Question 7. (**8+8=16-Points**) Let $\{X(t); t \geq 0\}$ be a geometric Browanian motion process with drift parameter $\mu = 0.02$ year, and variance parameter $\sigma^2 = 0.09$ year^2, where $X(t)$ represents the price of a stock (S) at time t. Given that the current price is \$1.75

(a) What is the probability that the price will be at least \$2.25 six month from now?

(b) What is the probability that the stock price will lose at most \$0.50 from its current value in the next six months?