KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 142

STAT416 : Stochastic Processes for Actuaries (142)

First Exam **Sunday March 8, 2015**

Name: **ID:**

Question Number	Full Mark	Marks Obtained
One	13	
Two	8	
Three	8	
Four	15	
Five	10	
Six	12	
Seven	14	
Total	80	

Question.1 (**2+2+2+2+5=13-Points**)

(a). Assume that we have a Markov Chain with states $0, 1, \ldots, i, \ldots, j, \ldots, n$, then define the following:

i. Accessible state:

ii. Irreducible Markov Chain

iii. Aperiodic state

iv. Ergodic state

(b). A markov chain $\{X_n, n \geq 0\}$ with states $0, 1$ and a transition probability matrix $\boldsymbol{P} = \begin{pmatrix} \frac{1}{6} & \frac{5}{6} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$.

Given that $P(X_0 = 0) = P(X_0 = 1) = \frac{1}{2}$, find $P(X_2 = 1)$.

Question.2 **(2+6=8-Points)** Consider the following Markov chain with transition probability matrix given by

$$\boldsymbol{P} = \begin{pmatrix} 0.5 & 0 & 0.5 & 0 & 0 \\ 0.25 & 0.5 & 0.25 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0.5 \end{pmatrix}. \text{ Then}$$

(a.) Specify the classes,the recurrent and transient states

(b.) Find the the period of the recurrent states

Question 3. **(8-Points)** A Markov chain $\{X_n, n \geq 0\}$ with transition probability matrix $\boldsymbol{P} = \begin{pmatrix} 0.1 & 0.1 & 0.8 \\ 0.2 & .2 & .6 \\ .3 & .3 & .4 \end{pmatrix}$, find $f_{02}^{(3)}$ by summing over all paths. (Hint: $f_{ij}^{(n)} = P\left(X_n = j, X_k \neq j, k = 0, 1, \ldots, n-1 | X_0 = i\right)$)

Question 4. **(10+5=15-Points)** Consider a Markov chain whose transition probability matrix is $\boldsymbol{P} = \begin{pmatrix} 0.4 & 0.6 \\ 0.7 & 0.3 \end{pmatrix}$.

Given that the eigne values are $1, 0.3$, and their corresponding eigne vectors are $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ \frac{3}{4} \end{pmatrix}$

(a). Find $\boldsymbol{P}^n$

(b) If the intial vector is $(1, 0)$ find $\lim_{n\to\infty} \boldsymbol{P}^{(n)}$

Question 5. **(2+2+2+4=10-Points)** Let $\beta(z)$ be the probability generating function for a branching process, where

$$\beta(z) = \frac{3}{4} + \frac{1}{8}z + \frac{1}{8}z^2,$$
$$(\beta(z))^2 = \frac{1}{4} + \frac{3}{8}z + \frac{17}{64}z^2 + \frac{3}{32}z^3 + \frac{1}{64}z^4,$$
$$\beta(\beta(z)) = \frac{23}{32} + \frac{3}{16}z + \frac{41}{512}z^2 + \frac{3}{256}z^3 + \frac{1}{512}z^4.$$

Find the following:

(a). $P(X_1 = 1 | X_0 = 1)$

(b). $P(X_1 = 3 | X_0 = 2)$

(c). $P(X_2 = 2 | X_0 = 1)$

(d) Find probability of extinction given that we started with one individual. (Hint: Find π_0)

Question 6. (**5+7=12-Points**) Consider the following Markov chain with states $0, 1, 2$ and transtion probability matrix given by $\boldsymbol{P} = \begin{pmatrix} 0.3 & 0.3 & 0.4 \\ 0.1 & 0.5 & 0.4 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}$,

(a). Is this Markov chain reversible? Explain.

(b). Find the limiting probability vector ($\boldsymbol{\pi}$).

Question 7. (**14-Points**) Consider a gambler whose starts with \$i, and at each play of the game has the probability p of wining \$1 and probability $q = 1 - p$ of losing \$1. Assuming that successive plays of the game are independt, what is the probability the gambler will reach \$ N before reaching \$0. (for $p \neq q$). (Huint: use difference equations)