Department of Mathematics and Statistics
Semester 142

STAT310 Second Major Exam April 22, 2015

Name: _____ ID #: _____

1) Consider the model

$$\boldsymbol{y = X_1\beta_1 + X_2\beta_2 + \varepsilon,}$$

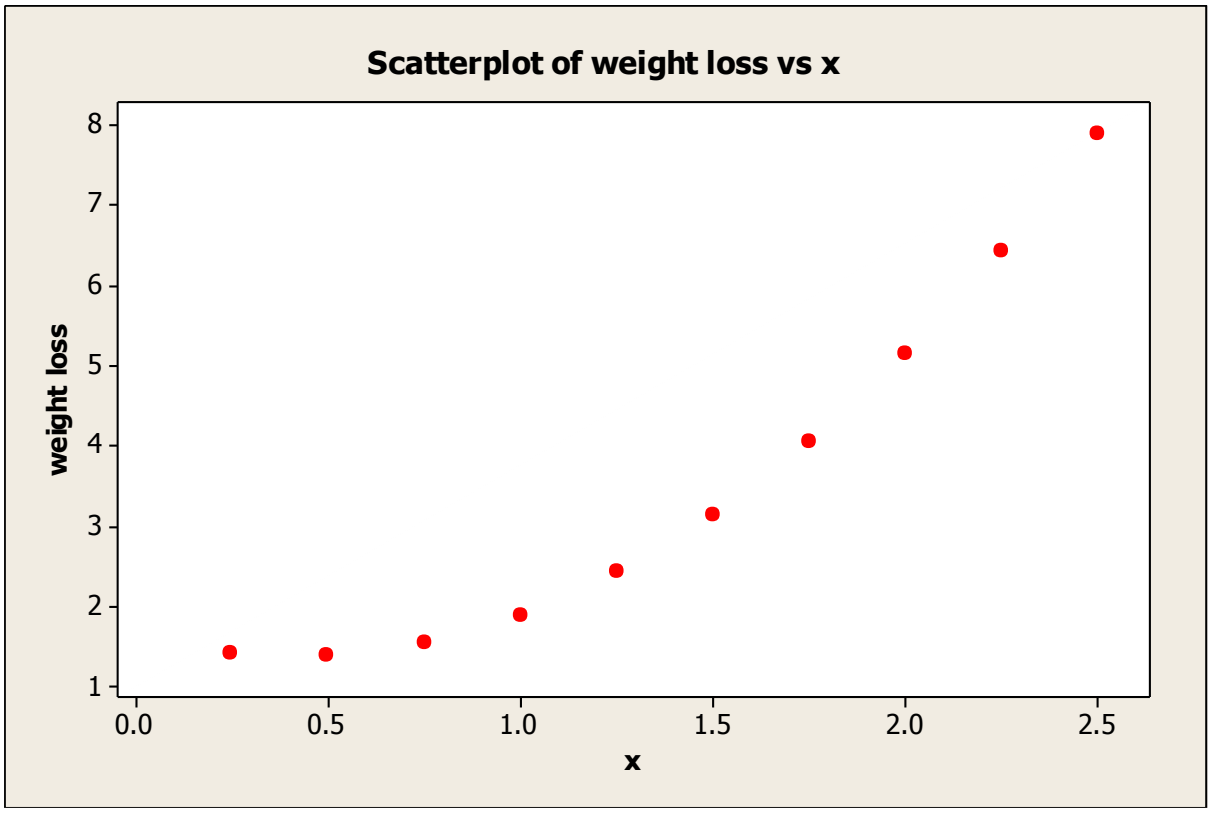
where $\boldsymbol{\beta_1}$ *is* $p_1 \times 1, \boldsymbol{\beta_2}$ is $p_2 \times 1$, $\boldsymbol{E(\varepsilon) = 0}$, $\boldsymbol{Var(\varepsilon) = \sigma^2 V}$, with σ^2 and $\boldsymbol{V}$ known.

Derive an appropriate test statistic for the hypothesis

$$H_0: \boldsymbol{\beta_2 = 0}, \quad \text{vs.} \quad H_1: \boldsymbol{\beta_2 \neq 0}$$

2) A solid-fuel rocket propellant loses weight after it is produced.

A scatter plot of the data is given below.



a) **Interpret the scatter plot.**

b) Fully interpret the first order model shown below.

Model I.

```
Predictor      Coef  SE Coef      T      P
Constant   -0.4220   0.5184  -0.81  0.439
x           2.8778   0.3342   8.61  0.000


S = 0.758866   R-Sq = 90.3%   R-Sq(adj) = 89.0%


Analysis of Variance

Source          DF      SS      MS      F      P
Regression       1  42.703  42.703  74.15  0.000
Residual Error   8   4.607   0.576
Total            9  47.310
```

c) Fully interpret the second order model shown below

Model II.

Coefficients

Term	Coef	SE Coef	T	P
Constant	1.63300	0.0041960	389.184	0.000
x*x	1.49455	0.0024841	601.642	0.000
x	-1.23218	0.0070096	-175.784	0.000

S = 0.00356753 R-Sq = 100.00% R-Sq(adj) = 100.00%
PRESS = 0.000220200 R-Sq(pred) = 100.00%

Analysis of Variance

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	2	47.3102	47.3102	23.6551	1858613	0.0000000
x*x	1	46.9169	4.6069	4.6069	361974	0.0000000
x	1	0.3933	0.3933	0.3933	30900	0.0000000
Error	7	0.0001	0.0001	0.0000		
Total	9	47.310				

3) Compare the two models.

4) Are there any potential hazards in extrapolating? Explain.

5) Consider the model $\boldsymbol{y = \beta_0 + \beta_1 x + \varepsilon}$, where $\boldsymbol{Var(\varepsilon) = \sigma^2 x^2}$.

i) Find an appropriate transformation to homogenize the variance.

ii) Write the model in terms of the new variables.

iii) After you fit the new model, explain how to find the fitted model in terms of the original variables.

iv) If you wanted to use weighted least squares (WLS), what quantity will you minimize?

v) Show that the WLS procedure used above is equivalent to ordinary least squares (OLS) on the transformed variables.