Confidence Interval Estimation – Two-Sample Problem

1. 100(1 – **α**)% Confidence Interval for the difference in the means $\mu_1 - \mu_2$ using two independent samples.

<u>Normal Populations or Large Samples</u>

If σ_1 and σ_2 are known: $$\left(\bar{x}_1 - \bar{x}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

If σ_1 and σ_2 are unknown: $$\left(\bar{x}_1 - \bar{x}_2\right) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

2. 100(1 – **α**)% Confidence Interval for the difference in the means $\mu_1 - \mu_2$ using <u>two independent small samples</u>

Two Normal Populations with <u>unknown</u> equal variances, $\sigma_1^2 = \sigma_2^2$,

$$\left(\bar{x}_1 - \bar{x}_2\right) \pm t_{\frac{\alpha}{2},\, f} \; s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}},$$

where $$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

and the number of degrees of freedom $f = n_1 + n_2 - 2$.

3. 100(1 – **α**)% Confidence Interval for the difference in the means $\mu_1 - \mu_2$ using <u>two independent small samples</u>

Two Normal Populations with <u>unknown</u> equal variances,

$$\left(\bar{x}_1 - \bar{x}_2\right) \pm t_{\frac{\alpha}{2},\, v}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Where the number of degrees of freedom

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

4. 100(1 – **α**)% Confidence Interval for the difference in the means of two related populations

$$\bar{d} \pm t_{\frac{\alpha}{2},\, n-1} \frac{s_d}{\sqrt{n}}$$

Where $\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$, $d_i = x_{1i} - x_{2i}$, and $s_d = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1}}$

5. Large Samples $(1-\alpha)100\%$ Confidence Interval for the difference between two population proportions, $\pi_1 - \pi_2$

$$(p_1 - p_2) \pm z_{\alpha/2} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Where $p_1 = \frac{x_1}{n_1}$, $p_2 = \frac{x_2}{n_2}$ are the sample proportions.

Assumptions:

1. $n_1 \pi_1 \geq 5$, $n_1(1-\pi_1) \geq 5$
2. $n_2 \pi_2 \geq 5$, $n_2(1-\pi_2) \geq 5$