King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 605 Homework # 1

Problem 1 *Use the Laplace method to verify the given symptotic approximations as* $x \to \infty$

$$(i) \qquad \int_0^{\frac{\pi}{4}} e^{x\cos t}\cos(nt)dt \sim \left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^x \quad , \quad n \text{ an integer.}$$

$$(ii) \qquad \int_{-1}^{1} e^{-x\cosh t}dt \sim \left(\frac{2\pi}{x}\right)^{\frac{1}{2}} e^{-x} \quad .$$

Problem 2 *Use Laplace's technique or, in some cases, watson's lemma, to obtain the first two terms in the asymptotic expansion as* $x \to \infty$ *of the following integral,*

$$I(x) = \int_0^{\infty} e^{-xt}\cos(t)dt$$