King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 605
Final Exam Part I (in class)
Duration: 2h
May 25, 2015

Name:.....ID:.....

Problem #1: (25 pts) Use the Laplace 's method to obtain an asymptotic approximation of the integral

$$f(x) = \int_0^1 \exp\left(-x\sqrt{t}\right) \frac{\cos t}{\sqrt{t}} dt, \qquad x \longrightarrow +\infty$$

Problem #2: (25 pts) Use the method of stationary phase to obtain the leading asymptotic behaviour as $x \longrightarrow +\infty$ of

$$g(x) = \int_0^1 \cos\left\{\lambda(t^3 - t)\right\} dt$$

Problem #3: (50 pts) Use the method of multiple scales to find the first two terms of an asymptotic expansion of the solution of the initial value problem

$$\begin{cases} \frac{d^2y}{dt} + 2\epsilon\frac{dy}{dt} + y = 0 \\ y(0) = 3 \ , \ y'(0) = 0 \end{cases}$$

when $\epsilon > 0$ is a small parameter, describing how the secular terms arise in the process and are suppressed to obtain a uniformly valid approximation.