King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 605 Final Exam (Part 2: Take home)
to hand in during the written part of the final

Important note: You are requested to provide a detailed account of the methods involved.

Problem #1: Use the WKB method to find asymptotic approximations of the eigenvalues μ^2 and corresponding eigenfunctions for large μ^2 for the following eigenvalue problem

$$\begin{cases} -\frac{d^2y}{dx^2} = \mu^2 x^2 y \ , \ 1 < x < 2 \\ y(1) = 0 \ , \ y(2) = 0 \end{cases}$$

Problem #2: Use matched asymptotic expansion of order n to find an approximation to the solution of the boundary value problem

$$\begin{cases} \epsilon\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0 \ , \ 0 < x < 1 \\ y(0) = 0 \ , \ y(1) = 2 \end{cases}$$

when $\epsilon > 0$ is small.

Plot on the same scale the outer solution, the inner solution, the approximate solution as well as the exact solution for $\epsilon = 0.01$