Math592/Game Theory & Applications
HomwWork 4: Equilibrium Refinements
Four Questions due May 4^{th}, 2015 [1]

Slim Belhaiza [2]

1 Regular Equilibrium?

What is a Regular Equilibrium? Are all Nash equilibria Regular? Give a small example.

2 Nash equilibrium & pure best replies

For a Nash equilibrium $\hat{X} = (\hat{X}_1, ..., \hat{X}_n)$ of a polymatrix game $G[(A_{ij})_{i \neq j}]$ with n players, let

$$M_i = M_i \left[A_{i.}, (\hat{X}_1, ..., \hat{X}_{i-1}, \hat{X}_{i+1}, .., \hat{X}_n) \right] = M_i \left[A_{i.}, \hat{X}_{-i} \right]$$

be the set of pure best replies of player i against $\hat{X}_{-i} = (\hat{X}_1, ..., \hat{X}_{i-1}, \hat{X}_{i+1}, ..., \hat{X}_n)$:

$$M_i = \left\{ \underset{h \in \{1,...,m_i\}}{\text{argmax}} \ (e_i^h)^T \sum_{j \neq i} A_{ij} \hat{X}_j \right\},$$

where e_i^h is a column vector with all entries equal to zero, except the h^{th} entry equal to one. Let us also define $CM_i = conv\,(M_i)$. Where $conv(M_i)$ indicates the convex envelope of the set M_i.

Show that $\hat{X}$ is a Nash equilibrium of a polymatrix game if and only if $\hat{X}_i \in CM_i$, for every player i.

3 Perfect equilibrium?

Selten's definition of perfect equilibrium for a strategic form game can be stated as follows for a bimatrix game.

Definition 3.1 *Let $\hat{X} = (\hat{X}_1, \hat{X}_2)$ be a Nash equilibrium of a bimatrix game. The equilibrium $\hat{X}$ is perfect if there exists a sequence $\{X^r\}_{r \in \mathbb{N}} = \{(X_1^r, X_2^r)\}_{r \in \mathbb{N}}$ of completely mixed strategy vectors converging to $\hat{X} = (\hat{X}_1, \hat{X}_2)$, such that for all $r \in \mathbb{N}$ and $i = 1, 2$:*

$$\hat{X}_i \in CM_i \left[A_i, X_{-i}^r \right].$$

Show that in every perfect equilibrium, for any given player, any weakly dominated strategy should be assigned a zero probability.

[1] This is NOT a team assignment. Make sure that you submit your answers individually using your own words.
[2] Department of Mathematics and Statistics, KFUPM: slimb@kfupm.edu.sa

4 Tribal Conflict Revisited

Recall the tribal fishing conflist we studied few weeks ago. Centuries ago, two tribes (1) and (2) settled on the two opposite shores of a desert's lake. The two tribes have always lived peacefully even if they shared a limited amount of resources consisting mainly in "fish". However, the population of each tribe increased as well as the average life expectancy. At the same time, each tribe has improved its fishing material and techniques. Due to these factors, a conflict emerged on the amounts of "fish"' each tribe has the right to fish every season. The local government has taken the initiative to send an independent emissary to lead the negotiation talks between the two tribes chiefs. With the help of some experts in renewable living resources, the emissary proposed an agreement on the quotas of "fish" allowed every fishing season. The proposed agreement guarantees the preservation of a minimum stock of fish in the lake and offers an acceptable and fair revenue to each tribe. After consultation with their tribal assembly members, each tribe's chief would give his tribe's final decision on the proposed agreement. Each tribe can accept **(A)** or reject **(R)** the proposed agreement, but its real intentions can only be verified at the end of the fishing season.

In the payoff matrices labeled "Tribal Conflict", tribe (1) chooses a row and tribe (2) chooses a column. In each cell, the first number represents the payoff to (1) and the second number represents the payoff to (2).

	Tribe (2) $\rightarrow$	
Tribe (1) $\downarrow$	**R**	**A**
R	-100, -100	50, 5
A	5, 50	10, 10

Table 1: Tribal Conflict

(a) Which of the extreme Nash equilibria of this game is proper? Explain.
(b) For each proper equilibrium identified in (a) find an adequate ϵ-proper equilibria sequence.

Suppose that the emissary can now recommend a strategy to each tribe.
(c) Is any of the extreme Nash equilibria correlated? Explain.
(d) If possible, find some other correlated equilibria of this game, or show that the game has **no** Non-Nash correlated equilibria.
(e) Discuss you results in (d): Do Correlated equilibria provide better payoffs for the tribes? Can we have better solutions to our conflict if the emissary was authorized to recommend strategies to the tribes?