**Math592/Game Theory & Applications**
**HomeWork 3: Nash Equilibrium Computation. Part II.**
Three Questions due April $6^{th}$, 2015 [1]

# 1 Subgame-perfect equilibrium? (10 points)

What is a subgame-perfect equilibrium? Is a subgame-perfect equilibrium sequentially rational? Give an example of an extensive form game with imperfect information such that the enumeration algorithm E$\chi$Mip finds a subgame-perfect equilibrium **only** when the sequential representation is used.

# 2 Move Game (20 points)

Two chain stores A and B are to enter in fierce competition. First, A has to choose whether to get "IN" or to stay "OUT" of the market zone served by the chain B stores. If A chooses to stay "OUT" the game ends, and the payoffs are as follows; A gets 2, and B gets 0. If A chooses to get "IN" then B observes this and has to choose whether to get "in" or to stay "out" of the market zone served by the chain A stores. If B chooses "out" the game ends, and the payoffs are; B gets 2, and A gets 0. If A chooses "IN" and B chooses "in" the game ends, and the payoffs are; B gets 3, and A gets -1.
(a) Draw a tree representing this game.**(4 points)**
(b) Find a subgame-perfect Nash equilibrium (SPNE) for this game. Detail how did you get your equilibrium.**(8 points)**
(c) Find all Nash equilibria for the reduced representation of this game. Which of these equilibria is a SPNE? Explain.**(8 points)**

# 3 A modified Card Game (20 points)

Recall the card game that you studied previously. The game was modified such that the shuffled deck contains 20 red cards and 32 black ones. The game is played by two people, whom we call "player 1" and "player 2". At the beginning of this game, each player puts 1 dollar in the pot. Next, player 1 draws a card from the shuffled deck. Player 1 looks at his card privately and decides whether to raise or to fold. If player 1 folds then he shows the card to player 2 and the game ends; in this case, player 1 takes the two dollars in the pot if the card is red, but player 2 takes the two dollars if the card is black. If player 1 raises then he adds another dollar to the pot and player 2 must decide whether to meet or pass. If player 2 meets, then she also must add another dollar to the pot, and then player 1 shows the card to player 2 and the game ends; in this case, again, player 1 takes the dollars in the pot if the card is red, and player 2 takes the dollars if the card is black. If player 2 passes, the game ends and player 1 gets the money in the pot.
(a) Find the extreme Nash equilibria of the sequential form representation of this card game. **(10 points)**
**Hint:** You can use the XGame Software or the Excel spreadsheet model available on the Blackboard.
(b) Is (are) the equilibrium (ia) found subgame-perfect? Detail your answer. **(10 points)**

[1] This is NOT a team assignment. Make sure that you submit your answers individually using your own words.