

1 Sequential Rationality? (10 points)

What is sequential rationality? What is backward induction? Does backward induction find a sequentially rational Nash equilibrium for every extensive game?

2 Road Trip (20 points)

Six students are going on a foreign trip on which they will live close together. Where they are going, there is a disease which spreads easily among people who live close together. The value of the trip to a student who does not get the disease is 6. The value of the trip to a student who gets the disease is 0.

There is a vaccination against the disease. The vaccination costs different amounts for different students (perhaps they have different health plans). Let's call the students 1; 2; 3; 4; 5 and 6 respectively. The vaccination costs 1 for student 1; it costs 2 for student 2; etc.... If a student gets vaccinated, he will not get the disease. But, if he is not vaccinated then his probability of getting the disease depends on the total number in the group who are not vaccinated. If he is the only person not to get vaccinated then the probability that he gets the disease is $1/6$. If there is one other person who is not vaccinated (i.e., two in all including him) then the probability that he gets the disease is $2/6$. If there are two other people who are not vaccinated (i.e., three including him) then the probability that he gets the disease is $3/6$, etc... For example, suppose only students 2 and 4 get vaccinated. Then 2's expected payoff is $6 - (2)$ where the 2 is the cost of the vaccination. Student 4's expected payoff in this case is $6 - (4)$. Student 5's expected payoff in this case (recall he did not get vaccinated) is $6 \times \frac{2}{6} + 0 \times \frac{4}{6} = 2$ where the fraction $\frac{4}{6}$ is the probability that he gets the disease. To model this game, suppose that each student aims to maximize his expected payoff.

The students decide, individually and simultaneously, whether or not to get a vaccination.

(a) Explain concisely whether or not it is a Nash equilibrium for students 1, 2, 3 and 4 to get vaccinated and students 5 and 6 not to get vaccinated. **(5 points)**

(b) Explain concisely whether or not it is a Nash equilibrium for students 1, 2 and 3 to get vaccinated and students 4, 5, and 6 not to get vaccinated. **(5 points)**

(c) Which players in this game have strictly or weakly dominated strategies? Explain your answers carefully including whether any domination is strict or weak. **(5 points)**

(d) If we delete all strictly and weakly dominated strategies from all players, which players in the game now have (iteratively) strictly or weakly dominated strategies? Explain carefully. **(5 points)**

3 Lend Money with No Regret (20 points)

Ali is deciding whether or not to lend money as a loan to his best friend Badr. Badr is poor and has a bad credit history. Badr has to decide whether or not to buy new furniture for his house. If he buys the furniture, he will be unable to repay the loan. If he does not buy, he will repay the loan. The payoffs in this game are as follows: if Ali refuses to lend money to Badr and Badr buys the furniture using a high interest bank loan, then Ali gets 0 and Badr gets -2 . If Ali refuses to lend money to Badr and Badr does not buy, then Ali and Badr get 0. If Ali lends money to Badr and Badr buys, then Ali gets -2 and Badr gets 7. If Ali lends money to Badr and Badr does not buy, then Ali gets a payoff of 3 and Badr gets a payoff of 5.

(a) Suppose this game is played simultaneously. Use the Lemke & Howson algorithm to find a Nash equilibrium for this game. **(8 points)**

(b) Suppose this game is played sequentially with perfect information and Ali plays first. Draw the game's tree. Use backward induction to find a Nash equilibrium for this game. **(6 points)**

(c) Suppose this game is played sequentially with perfect information and Badr plays first. Draw the game's tree. Use backward induction to find a Nash equilibrium for this game. **(6 points)**