King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
Math592/Game Theory & Applications
HomeWork 1
Three Questions, due March 2^{nd}, 2015

1 Short Questions (6 points)

State whether each of the following statements is true or false. For each, explain your answer in (at most) a short paragraph, example or counter-example.

(a) A weakly dominated strategy can never be a best response.

(b) Strategic form is the most complete way to model conflict situations.

(c) A Nash equilibrium is a situation where every player gets always his or her absolute maximum payoff.

2 Party Game (20 points)

Player A has invited player B to his party. Player A must choose whether or not to hire a clown. Simultaneously, player B must decide whether or not to go to the party. Player B likes A but hates clowns (he even hates other people seeing clowns!) B's payoff from going to the party is 4 if there is no clown, but 0 if there is a clown there. B's payoff from not going to the party is 3 if there is no clown at the party, but 1 if there is a clown at the party. A likes clowns (he especially likes B's reaction to them) but does not like paying for them. A's payoff if B comes to the party is 4 if there is no clown, but $8 - x$ if there is a clown (x is the cost of a clown). A's payoff if B does not come to the party is 2 if there is no clown, but $3 - x$ if there is a clown there.

(a) Write down the payoff matrices of this game **(4 points)**.

(b) Suppose $x = 0$. Identify all dominated strategies. Explain. Find a Nash equilibrium. What are your equilibrium payoffs?**(6 points)**

(c) Suppose $x = 2$. Identify all dominated strategies. Explain. Is there any Nash equilibrium in pure strategies? Find a Nash equilibrium.**(6 points)**

(d) For which values of x player A would always avoid hiring a clown independently from player B's choice? Explain.**(4 points)**

3 Four Prisoners (24 points)

Four prisoners, A, B, C and D, are sentenced to death and held, respectively, in four separate cells 1, 2, 3 and 4. The prisoners are to be executed at 06 : 00 am. The night of the execution, the governor has selected one cell number at random and its occupant, at 05 : 00 am, is to be pardoned. At 04 : 00 am exactly, the governor's office communicates the chosen cell's number to the prison's commander. At 04 : 10 am, a guardian comes to see prisoner A. The guardian knows which prisoner is pardoned and also knows that prisoner D passed away late that night (i.e., cell number 4 is now empty), but he is not allowed to tell. Prisoner A offers a bribe to the guardian to let him know the cell's number of one of the others who is going to be executed. Prisoner A says "If 2 is to be pardoned, give me number 3 or 4. If 3 is to be pardoned, give me number 2 or 4. If 4 is to be pardoned, give me number 2 or 3. And if I am to be pardoned, give me number 2, 3 or 4."
Prisoner A assumes that the four cells have the same probability of being selected by the governor. He also assumes that the guradian assigns the same probability to the cells he has to choose from. The guardian accepts the bribe but decides, on his own, to never give number 4 and to flip a coin in case he has to choose between 2 or 3. Moreover, the guradian demands an additional bribe from prisoner A if he requests his help to be moved, secretly, to cell number 4.

(a) Draw the game tree illustrating prisoner A's problem. **(10 points)**

(b) Do you think that prisoner A would increase his chances to be pardoned if he is moved to cell 4? Explain. **(10 points)**

(c) If prisoner A knew that the guardian would never give him number 4, do you think that he would change his mind? Explain. **(4 points)**