King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
Math592/Game Theory & Applications
Final Exam
Four Questions, May 16^{th}, 2015 [1]

1 Short Questions (24 points)

State whether each of the following statements is true or false **(2 point)**. For each, explain your answer in (at most) a short paragraph, example or counter-example **(3 points)**.

(a) A perfect Nash equilibrium is always proper. (6 **points**)

(b) Every Correlated equilibrium is a Nash equilibrium. (6 **points**)

(c) Finite and infinite repeated games always have the same Nash equilibria set. (6 **points**)

(d) A player using a trigger strategy initially cooperates but punishes the opponent if a certain level of defection is observed. (6 **points**)

[1] The exam lasts 120 minutes.

2 Correlated equilibrium and Chicken Game (26 points)

Consider a version of the "Chicken game" described as follows. Like all forms of the chicken game, there are three Nash equilibria. The two pure strategy Nash equilibria are (D, C) and (C, D). There is also a mixed strategy equilibrium which results in expected payoffs of $14/3 = 4.667$ for each player.

	D	C
D	$0, 0$	$7, 2$
C	$2, 7$	$6, 6$

Now consider a third party (or some natural event) that draws one of three cards labeled: (C, C), (D, C), and (C, D). This exogenous draw event is assumed to be uniformly at random over the 3 outcomes. After drawing the card the third party informs the players of the strategy assigned to them on the card (but not the strategy assigned to their opponent).

(a) Suppose a player is assigned D. Supposing the other player played their assigned strategy, would he want to deviate? **(7 points)**

(b) Suppose a player is assigned C. Supposing the other player played their assigned strategy, would he want to deviate? **(7 points)**

(c) Find the third Nash equilibrium in mixed strategies. Does it represent a coordinated equilibrium of this game? **(12 points)**

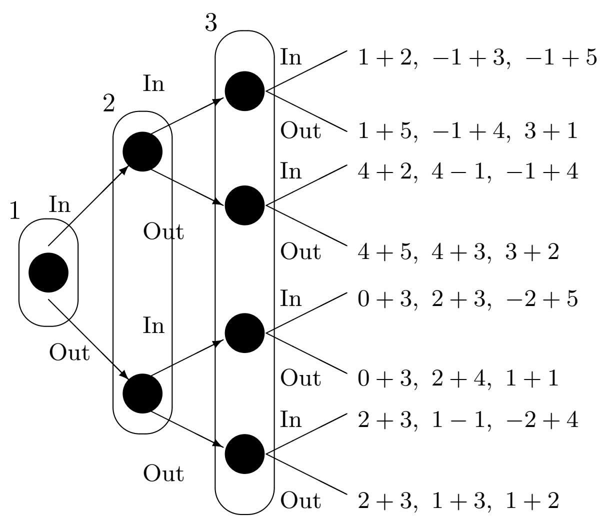
3 Chain store competition game (26 points)

Figure 1 illustrates an extensive competition game with imperfect information involving three chain stores. Each of the chain stores 1, 2 and 3, has to decide either to enter the market zones of both of its opponents or not. Hence, each chain store randomizes on two pure strategic decisions "In" and "out". Each chain store gets a partial payoff depending on its decision and the opponents' decisions. For example, if chain store 1 decides to get "In" while chain stores 2 and 3 decide to stay out, chain store 1 gets $4+5$ as a total payoff and chain stores 2 and 3 get, respectively, $4+3$ and $3+2$. This game can be reduced to a three-person polymatrix game with the following payoff matrices.

$$A_{1\cdot} = \left(\begin{array}{cc|cc} 1 & 4 & 2 & 5 \\ 0 & 2 & 3 & 3 \end{array} \right) \qquad A_{2\cdot} = \left(\begin{array}{cc|cc} -1 & 2 & 3 & 4 \\ 4 & 1 & -1 & 3 \end{array} \right)$$

$$A_{3\cdot} = \left(\begin{array}{cc|cc} -1 & -2 & 5 & 4 \\ 3 & 1 & 1 & 2 \end{array} \right)$$

Figure 1: Chain Store Competition



(a) Using the polymatrix reduction of this game, formulate the utility maximization program for each chain store $i = 1, 2, 3$. **(12 points)**

(b) Assuming now perfect information, find a subgame perfect equilibrium for the original extensive game. **(14 points)**

4 Vehicle Routing Game (24 points)

A customer i bought some merchandise from a mega-store. The customer is waiting for it to be delivered to his home. On the given delivery day, he is part of a group of n customers scheduled to be visited by the p drivers of the transportation company. Suppose that the item(s) customer i purchased can be represented by a demand parameter D_i. Suppose also that the total capacity of a driver's truck can be represented by a capacity parameter C_j. The utility of customer i, for being served by driver j, is p_{ij}. The utility of driver j, for serving customer i, is q_{ij}. Suppose that every customer can be served by more than one driver. Suppose also that every truck can serve many customers. Finally, suppose that every customer and every driver can decide, respectively, which driver is serving him, and which customer(s) to serve. In the following, you are asked to model this problem as a strategic form game.

(a) Define the decision variable(s) for each customer i. **(6 points)**

(b) Define the decision variable(s) for each driver j. **(6 points)**

(c) Formulate the utility maximization program for each customer i. **(6 points)**

(d) Formulate the utility maximization program for each driver j. **(6 points)**