

**Math592/Game Theory & Applications**  
**Midterm Exam**  
April 8<sup>th</sup>, 2015 <sup>1</sup>

Name: \_\_\_\_\_ <sup>2</sup>

## 1 Perfect vs Imperfect Information Game (25 points)

Consider a game involving two players. Player 1 plays once or twice while player 2 plays at most once. For his first move, player 1 has two alternatives **a** or **b**. After player 1 decides, player 2 will have two alternatives **c** or **d**.

If player 1 played **a** and player 2 plays **c**, player 1 decides to play **e**, **f** or **g**, and both players receive respectively (3,2), (1,3) or (2,1) as final payoffs. While if player 1 played **a** and player 2 plays **d**, the game ends and both players receive (0,1) as final payoffs. If player 1 played **b** and player 2 plays **c**, player 1 decides to play **e**, **f** or **g**, and both players receive respectively (1,3), (2,2) or (0,1) as final payoffs. While if player 1 played **a** and player 2 plays **d**, the game ends and both players receive (2,0) as final payoffs.

- (a) Draw a tree representing this game. (7 points)

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<sup>1</sup>The exam game last for 90 minutes.

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(b) Assuming perfect information, use backward induction to find a sequentially rational Nash equilibrium. **(8 points)**

(c) Assuming imperfect information, find the extreme Nash equilibria of the reduced and sequential representations. Which equilibrium is subgame-perfect? **(10 points)**

## 2 Smugglers vs Customs (25 points)

An import-export business company (X) is operating in the borders region between two countries. The company ships containers from country  $A$  to country  $B$ . The containers are loaded on large trucks and travel long distances to pass the borders. Due to the high levels of economic borders' activity between the two countries, the customs officers (Y) of the two countries perform common coordinated random inspections of the merchandise loaded in each container. The company (X) has the opportunity to realize huge profits if it succeeds in shipping large quantities of a precious metal to country  $B$ . This illegal but very profitable activity is called **smuggling**.

If the customs (Y) do inspect and catch the company (X) smuggling, they impose a large penalty payment  $P > 0$  for this crime. The company (X) has to choose between two strategies; Ship (S) or Not-Ship (NS). The customs have to choose between two strategies; Inspect (I) or Not to inspect (NI). If the customs play (I) and the company plays (S), the customs get a payoff of  $P$  and the company gets a payoff of  $-P$ . If the customs play (I) and the company plays (NS), the customs get a payoff of  $-10$  and the the company gets a payoff of  $0$ . If the customs play (NI) and the company plays (S), the customs get a payoff of  $-100$  and the company gets a payoff of  $500$ . Finally, if the customs plays (NI) and the company play (NS), the customs get a payoff of  $0$  and the consumer gets a payoff of  $0$ .

(a) For which values of  $P$  none of the pure strategies of (X) and (Y) is strictly dominated by the other. (5 points)

(b) Assume that  $P = 1000$ . Use the **Lemke & Howson's algorithm** to find a Nash equilibrium of this game. (10 **points**)

(c) Assume that  $P = 500$ . Use the **Lemke & Howson's algorithm** to find a Nash equilibrium of this game. (10 **points**)