King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 572 , Term: 142 — Assignment (5)

Instructor: Dr. Faisal Fairag — Due Sunday 29/3/2015

---

(1) [Problem 5.7 Page 74]

---

(2) Consider the Neumann problem

$$-\nabla\cdot(a\nabla u)+cu=f,\ \text{in}\ \Omega$$
$$\frac{\partial u}{\partial n}=0\ \text{on}\ \partial\Omega$$

where $a(x)\ge a_0>0, c(x)\ge c_0>0$ Formulate a finite element problem and then prove error estimate.

---

(3) consider the 9-point approximation to the laplacian

$$-\Delta_{9pt}^{\theta}=-\left(\theta\,\Delta_h+(1-\theta)\,\Delta_{2h}\right)$$

a) Find the value of θ which makes $-\Delta_{9pt}^{\theta}$ $O(h^4)$ accurate.

b) Represent this as a stencil.

---

(4) consider the elliptic equation

$$-(au_{xx}+2bu_{xy}+cu_{yy})=f,\ \text{in}\ \Omega$$
$$u=g\ \text{in}\ \partial\Omega$$
$$\Omega=(0,1)\times(0,1)\quad b^2<ac$$

Discretize the mixed derivative by $\frac{\partial^2}{\partial x\partial y}\approx\partial_1\partial_2+\bar{\partial}_1\bar{\partial}_2$

a) What is the order of accuracy is attained

b) Under what conditions is this of positive type?

---

(5) In the proof of the Theorem 4.4 page 47, show the following

a) $Wi_j\ge 0$

b) $-\Delta_h W_{ij}=4$

c) $Wi_j\le 1/2$

d) $V_{ij}^{-}\le|U|_{\Gamma}$
