King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 572 , Term: 142 | Assignment (3)
Instructor: Dr. Faisal Fairag | Due Sunday 22/2/2015

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(1) Consider the problem $-((1+x)u')' = 0$, $\Omega = (0,1)$, $u(0) = 0, u'(1) = 1$
Divide the domain into three subintervals of equal length h=1/3 and let V_h be the corresponding space of continuous piecewise linear functions vanishing at x=0

(a) Use V_h to formulate a finite element method.

(b) Verify that the stiffness matrix A and load vector b are given by

$$A = \frac{1}{2}\begin{bmatrix} 16 & -9 & 0 \\ -9 & 20 & -11 \\ 0 & -11 & 11 \end{bmatrix}, \qquad b = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

(c) Verify that A is symmetric and positive definite.

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(2) consider the following problem: $-((x^2+2)u')' = \dfrac{x^2}{x^2+1}$ in $\Omega = (0,1)$ with the boundary condition $u(0) = u'(1) = 0$. Its weak form is: Find $u \in H$ such that $a(u,v) = L(v) \quad \forall v \in H$ where $a(u,v) = \int_0^1 (x^2+2)u'(x)v'(x)dx$ and $L(v) = \int_0^1 \dfrac{x^2}{x^2+1} v(x)dx$ and $H = \{v \in L_2(\Omega) : v' \in L_2(\Omega), v(0) = 0\}$.
Use Lax-Milgram lemma to show that there exist a unique solution.

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(3) [Problem 5.6 Page 74]

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(4) Modify the Matlab codes createK.m and main.m to solve the following problem

$$-u'' = f(x) \quad \text{with} \quad u(0) = u(1) = 0$$

where $f(x) = x^2$. Use the space of continuous pricewise quadratic functions.
Then approximate the value of u at x = 0.5 and plot the solution.
