

**(1)** Let  $\Omega = (0,1)$  and  $f(x) = x$   $g(x) = \cos(x)$ . Verify Cauchy-Schwarz inequality.

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**(2)** [Problem A.5 Page 240]

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**(3)** Give vibrational formulation for the problem:

$$-u'' = f \quad \text{with} \quad u'(0) = u'(1) = 0$$

and explain why this problem is not well-posed.

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**(4)** Modify the Matlab code FEM\_1D.m to solve the following problem

$$-u'' + b(x)u' + c(x)u = f(x) \quad \text{with} \quad u(0) = u(1) = 0$$

where  $b(x) = -x^3$   $c(x) = x^2$   $f(x) = (16\pi^2 + x^2)\sin(4\pi x) - 4\pi x^3 \cos(4\pi x)$

Then approximate the value of  $u$  at  $x = 0.5$  and plot the solution.

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