

Problem # 1. (7 marks) Solve the problem

$$\frac{u_x}{x} - \frac{u_y}{y} = 4u, \quad x \neq 0, \quad y \neq 0$$

$$u(x, x) = 1$$

Problem # 2. (14 marks) Use the characteristic method to find **two** solutions of

$$u_x u_y = u, \quad u(x, 2x) = \frac{9}{4}x^2$$

Problem # 3. (14 marks) Let

$$u_{xx} - 2e^y u_{xy} + e^{2y} (u_{yy} + u_y) = 0$$

- a. Show that the PDE is parabolic
- b. By a convenient change of variable, reduce it its canonical form
- c. Find the solution $u(x, y)$ and its domain of definition, if $u(0, y) = -y$ and $u_x(0, y) = 2e^y$.