King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 513 Assignemt 3 (Term 142)

Name : .... **ID** #...................................

Question 1: Find the solution $u = u(r, \theta)$ for the following boundary value problem:

$$r^2 u_{rr} + r u_r + u_{\theta\theta} = 0, \quad for \ \ 1 < r < 2, \quad 0 < \theta < \pi,$$

$$u(r, 0) = u(r, \pi) = 0, \quad for \ \ 1 < r < 2,$$
$$u(1, \theta) = \sin\theta, \quad and \ \ u(2, \theta) = 0, \quad for \ \ 0 < \theta < \pi.$$

Question 2: Find the temperature distribution $T = T(r, \theta)$ modeled by the boundary value problem:

$$r^2 T_{rr} + r T_r + T_{\theta\theta} = 0, \quad for \ \ 0 < r < c, \quad 0 < \theta < \pi,$$

$$T(r, 0) = T(r, \pi) = 0, \quad for \ \ 0 < r < c,$$
$$T(c, \theta) = 100.$$

Question 3: The three-dimensional, rectangular coordinates (x, y, z) are mapped to the spherical coordinates (r, θ, ϕ) by the relations

$$x = r\cos\theta\sin\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\phi.$$

Note also that

$$r^2 = x^2 + y^2 + z^2, \quad and \quad \tan\theta = \frac{y}{x}.$$

Let $u = u(x, y, z)$. Write the partial differential equation

$$-u_{xx} + u_x = 0$$

in the spherical coordinates.