Name : ...... ID #...... Serial #: .....

**Question 1**: Use the separation of variables to solve the following partial differential equations for u = u(x, t)

- 1.  $u_t + u_x = 2u$
- 2.  $u_{tt} u_{xx} = 0$

**Question 2**: Consider the function  $u = u(r, \theta)$  defined in polar coordinates as

$$u(r,\theta) = (c_1 cos(a\theta) + c_2 sin(a\theta))(c_3 r^a + c_4 r^{-a}),$$

where  $c_1, ..., c_4$ , and a are some parameters. Show that the function u is a solution of the partial differential equation:

$$r^2 u_{rr} + r u_r + u_{\theta\theta} = 0.$$

**Question 3**: Solve the following boundary value problems using laplace transform.

1. Problem: Find u = u(x, t) satisfying

$$u_{xx} = u_t, \quad 0 < x < 1, \quad t > 0$$
$$u(0,t) = u(1,t) = 0, \quad t > 0$$
$$u(x,0) = x(1-x)$$

2. Problem: Find u = u(x, t) satisfying

$$u_{xx} = u_{tt}, \quad 0 < x < 1, \quad t > 0$$
$$u(0,t) = u(1,t) = 0, \quad t > 0$$
$$u(x,0) = 0, \quad u_t(x,0) = sin(\pi x)$$