King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 513 Final (Term 142)

Name : .... **ID** #......................................

Question 1: Use Laplace transform to solve the initial value problem for $y = y(x)$,

$$y'' + 5y' + 6y = \frac{d}{dx}\left[3H(x-2) - 4H(x-5)\right], \quad x > 0,$$
$$y(0) = y'(0) = 0.$$

Question 2: Find the complex Fourier transform of the periodic function,

$$g(t) = e^t H(\pi t - t^2), \quad for \quad |t| < \pi.$$

Question 3: Solve the following integral equation

$$f(t) + \int_0^{2t} f\left(t - \frac{1}{2}x\right) \sinh x \; dx = t^2.$$

Question 4: Find the temperature distribution $T = T(r, \theta)$ modeled by the boundary value problem:

$$r^2 T_{rr} + rT_r + T_{\theta\theta} = 0, \quad for \ \ 0 < r < c, \quad 0 < \theta < \pi,$$

$$T(r, 0) = T(r, \pi) = 0, \quad for \ \ 0 < r < c,$$

$$T(c, \theta) = 100.$$

Question 5: The three-dimensional, rectangular coordinates (x, y, z) are mapped to the spherical coordinates (r, θ, ϕ) by the relations

$$x = r\cos\theta\sin\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\phi.$$

Note also that

$$r^2 = x^2 + y^2 + z^2, \quad \textit{and} \quad \tan\theta = \frac{y}{x}.$$

Let $u = u(x, y, z)$. Write the partial differential equation

$$-u_{yy} + u_y + u = x^2 + y^2$$

in the spherical coordinates.

Question 6: Derive the general solution $X = X(t)$ for the system of ordinary differential equations:

$$X'(t) = AX(t) + F(t),$$

$$X(0) = C.$$

where A is an $n \times n$ matrix, while $X(t)$ and $F(t)$ are $n \times 1$ matrices.