King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 513 Test 3 (Term 142)

Name : ..... **ID** #.....

Question 1[6 points]: Consider the function $u = u(r, \theta)$ defined in polar coordinates as

$$u(r, \theta) = [cos(\theta) + sin(\theta)](r + r^{-1}),$$

Show that the function u is a solution of the partial differential equation:

$$r^2 u_{rr} + r u_r + u_{\theta\theta} = 0.$$

Question 2[7 points]: Solve the following boundary value problem using the separation of variables: Find $u = u(x,t)$ satisfying

$$u_{xx} = u_t, \quad 0 < x < \pi, \quad t > 0$$

$$u(0,t) = u(\pi,t) = 0, \quad t > 0$$

$$u(x,0) = sin(x) + sin(3x)$$

Question 3[7 points]: Solve the following boundary value problem using Laplace transform: Find $u = u(x,t)$ satisfying

$$u_{xx} = u_{tt}, \quad x > 0 \quad 0 < t < 1,$$

$$u(x,0) = u(x,1) = 0, \quad t > 0$$

$$u(0,t) = 0, \quad u_x(0,t) = -sin(\pi x)$$