King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Instructor: Khaled Furati

MATH 411 - Exam 2 - Term 142

Duration: 120 minutes

Student Name:

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1. Define the following.
   (a) $f : \Omega \subseteq \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $a \in \Omega$.
   (b) $a \in \Omega$ is a degenerate critical point of $f : \Omega \subseteq \mathbb{R}^n \to \mathbb{R}$.
2. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Show that $D_T(a) = T$ for all $a \in \mathbb{R}^n$.
3. Let $f(x, y) = (x^2, x \sin y, e^{xy})$. Find $D_f(x, y)$.
4. Let $f(x, y) = (2x - y, xy)$ and $g(u, v) = (u - v, 2u + v)$. Use chain rule to find $D_{(f \circ g)}$.
5. Let f be a real-valued differentiable function on an open convex set Ω in $\mathbb{R}^n$. Show that if ∇f is bounded on Ω then f is Lipschitz on Ω.
6. Let $f(x, y) = x^2 y$ and $g(x, y) = x - y^2$. Let $h = (f, g)$. Compute $H_f(x, y)$ and $D_h^2(x, y)$.
7. Show that the function $f(x, y) = 2x + xy - x^2 y^4$ has a critical point but no local extrema.
8. Show that the equations
   $$\begin{aligned} xy^5 + yu^5 + zv^5 &= 1 \\ x^5 y + y^5 u + z^5 v &= 1 \end{aligned}$$
   have a unique solution $(u, v) = f(x, y, z) = (g(x, y, z), h(x, y, z))$.

Q#	**Points**	**Points**
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
Total		80