King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

Math 345 – Modern Algebra I (Spring 2015)

Major Exam 1 (Duration = 100 minutes)

Exercise 1 **[20 minutes = 20 points]**

(a) **[5 points]** Give a finite subgroup of $\mathbb{R}^*$ ($\neq \{1\}$).

(b) **[5 points]** Give an example of a group which has a subgroup that is not normal.

(c) **[5 points]** Give an example of a non-abelian group in which every subgroup is normal.

(d) **[5 points]** Give an example where the converse of Lagrange's theorem is false [**Describe the example without proof**].

Exercise 2 **[20 minutes = 20 points]**

(a) **[5 points]** Determine all possible generators of the cyclic group $\dfrac{\mathbb{Z}}{30\mathbb{Z}}$

(b) **[15 points]** Draw the lattice of all its subgroups (i.e., with all possible inclusions between subgroups and, for each subgroup, give its generator and order).

Exercise 3 **[20 minutes = 20 points]**

Let G be a group with $|G| = 51$. Let $a, b \in G \setminus \{1\}$ with $|a| \neq |b|$. Let H be a subgroup of G. Show: $a, b \in H \Longrightarrow H = G$.

Exercise 4 **[20 minutes = 20 points]**

Let $G := \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mid a, b \in \dfrac{\mathbb{Z}}{p\mathbb{Z}} \right\}$ where p is prime.

(a) **[10 points]** Show G is an abelian group (under multiplication) and find its order.

(b) **[10 points]** Is G cyclic?

Exercise 5 **[20 minutes = 20 points]**

Let A_8 denote the alternating group of order 8.

(a) **[10 points]** Let $\sigma \in A_8$ with $\sigma = \alpha_1 \alpha_2 \dots \alpha_k$ where the α_i's are disjoint cycles. Show that $k = 2\ or\ 4$.

(b) **[10 points]** What is the maximum order an element of A_8 can have?