Name: ID #: Section #:

(**1**) Evaluate $\displaystyle\int_C \frac{1+z}{z}\,dz$, C is the right half of the circle $|z|=1$ from $z=-i$ to $z=i$.

Solution: On C we have $z=e^{it}$, $0\le t\le 2\pi$.

$$\int_C \frac{1+z}{z}\,dz = \int_0^{2\pi} \frac{1+e^{it}}{e^{it}}\,ie^{it}dt = \int_0^{2\pi} i(1+e^{it})\,dt = (\pi+2)i$$

(**2**) Evaluate $\displaystyle\oint_C \left(\frac{e^z}{z+3} - 3\bar{z}\right) dz$, C is the unit circle $|z|=1$.

Solution: Since $\dfrac{e^z}{z+3}$ is analytic on and within C, we have

$$\int_C \frac{e^z}{z+3}dz = 0.$$

Thus,

$$\oint_C \left(\frac{e^z}{z+3} - 3\bar{z}\right) dz = -\oint_C 3\bar{z}\,dz = -\int_0^{2\pi} 3e^{-it}(ie^{it}dt) = -6\pi i$$

(**3**) Use Cauchy's integral formula to evaluate $\displaystyle\oint_C \left(\frac{\sin z}{(2z-\pi)^3} - \frac{e^{iz}}{z+i}\right)\,dz$, C is the circle $|z|=3$.

Solution: Let $f_1(z)=\frac{1}{8}\sin z$ and $f_2(z)=e^{iz}$. We have

$$\begin{aligned}\oint_C \left(\frac{\sin z}{(2z-\pi)^3} - \frac{e^{iz}}{z+i}\right)\,dz &= \oint_C \left(\frac{\sin z}{2^3(z-\pi/2)^3} - \frac{e^{iz}}{z+i}\right)\,dz \\ &= \frac{2\pi i}{2!}f_1''(\pi/2) + 2\pi i f_2(-i) \\ &= \pi i\left(-\frac{1}{8}\sin\pi/2\right) + 2\pi i e = -i\pi\left(\frac{1}{8}+2e\right)\end{aligned}$$