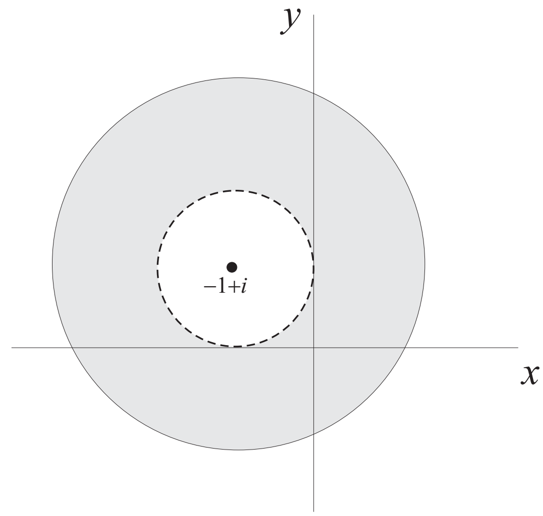
(**1**) Sketch the set of points satisfying the inequality $1 < |z + 1 - i| \le 3$. Determine whether the set is a domain.

Solution: The set is not a domain, since it is not an open set.




(**2**) (a) Find $\lim_{z\to i} \frac{z^4 - 1}{z - i}$

(b) Give the points at which the function $f(z) = \frac{z^3 + z}{z^2 + 1}$ is not analytic.

Solution:

(a) $\lim_{z\to i} \frac{z^4 - 1}{z - i} = \lim_{z\to i}(z^2 - 1)(z + i) = -4i$

(b) $f$ is not analytic when $z = \pm 2i$.

(**3**) Write the answers of the following questions in the form $a + ib$.

(a) Compute $z^9$, if $z = \frac{-1 + i\sqrt{3}}{1 + i}$.

(b) Find the three cube roots of $z = -1 + i$.

Solution:

(a) Since $z = \frac{-1 + i\sqrt{3}}{1 + i} = \sqrt{2}(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12})$, We have

$$z^9 = (\sqrt{2})^9\left(\cos\frac{15\pi}{4} + i\sin\frac{15\pi}{4}\right) = 16 - 16i$$

(b) In this case, $r = \sqrt{2}$ and $\theta = 3\pi/4$. We have

$$w_0 = (\sqrt{2})^{1/3}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 2^{1/6}(1/\sqrt{2} + i/\sqrt{2}),$$

$$w_1 = 2^{1/6}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right),$$

$$w_2 = 2^{1/6}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right).$$