King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Final Exam
The Second Semester of 2014-2015 (142)

Time Allowed: 180 Minutes

Name: ________________________ ID#: ______________

Instructor: ____________________ Sec #: ________ Serial #: ________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		12
2		22
3		22
4		22
5		22
6		20
7		20
Total		140

Q:1(8+4 points) Consider the Sturm–Liouville problem

$$y'' - \tan x y' + \lambda y = 0 \text{ with } y(\tfrac{\pi}{4}) = 0,\ y(\tfrac{\pi}{3}) = 0.$$

(a) Put the differential equation in self–adjoint form and write its weight function.

(b) If y_n and y_m are two eigenfunctions corresponding to two different eigenvalues, write the orthogonality relation.

Q:2 (22 points) Use separation of variables method to solve the problem

$$5\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < \pi, \ t > 0,$$

subject to the boundary and initial conditions

$$\begin{aligned} u_x(0,t) &= 0, \quad u_x(\pi,t) = 0, \quad t > 0, \\ u(x,0) &= x, \quad 0 < x < \pi. \end{aligned}$$

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Q:3 (22 points) Use Laplace transform to solve the problem

$$4\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad x > 0, \ t > 0,$$

subject to the boundary and initial conditions

$$\begin{aligned} u(0,t) &= f(t), \quad \lim_{x\to\infty} u(x,t) = 0, \quad t > 0, \\ u(x,0) &= 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \quad 0 < x < 1, \end{aligned}$$

where $f(t) = \begin{cases} \sin \pi t & 0 < t \leq 1 \\ 0 & t > 1 \end{cases}$

Q:4 (22 points) Use separation of variables method to solve the problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0,\ \ 0 < r < 2,\ \ 0 < z < 5,$$

subject to the boundary conditions

$$\begin{aligned} u\,(2, z) &= 0,\ \ 0 < z < 5 \\ u\,(r, 0) &= 0,\ \ 0 < r < 2 \\ u\,(r, 5) &= 4,\ \ 0 < r < 2 \end{aligned}$$

Also solution $u(r, z)$ is bounded at $r = 0$.

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Q:5 (22 points) Find the steady-state temperature $u(r, \theta)$ in a sphere of radius 2 by solving the problem

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta} = 0, \quad 0 < r < 2, \ 0 < \theta < \pi,$$

subject to the boundary condition

$$u(2, \theta) \quad = \quad 1 + \cos(\theta), \quad 0 < \theta < \pi.$$

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Q:6 (20 points) Find Fourier integral representation of

$$f(x) = \begin{cases} 0, & x < -1 \\ 2, & -1 < x < 0 \\ -2, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$$

Q:7 (20 points) Use appropriate Fourier transform to find the temperature $u(x,t)$ in a semi-infinite rod if $u_x(0,t) = 0$ and $u(x,0) = e^{-x}, \;\; x > 0$.