King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Major Exam 2
The Second Semester of 2014-2015 (142)
Time Allowed: 120 Minutes

Name: _____ ID#: _____

Instructor: _____ Sec #: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		15
2		15
3		12
4		12
5		10
6		10
7		14
8		12
Total		100

Q:1 (7+8 points) Find the following:

(a) $\mathcal{L}^{-1}\left\{\dfrac{1}{s(s+1)^2}\right\}$, by using convolution,

(b) Write $f(t) = \begin{cases} \sin(t), & 0 \le t < \pi \\ e^{2t}\cos(t), & t \ge \pi \end{cases}$

in compact form using unit step function and find its Laplace transform.

Q:2 (5+5+5 points) Find the following:

(a) $\mathcal{L}\{te^{-2t}\cos 3t\}$,

(b) $\mathcal{L}\{f(t)\}$, where $f(t) = \begin{cases} 2t+1, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$.

(c) $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+2s-3}\right\}$.

Q:3 (12 points) Solve the following boundary value problem using Laplace transform

$y'' + 9y = \cos 2t$ with $y(0) = 1$, $y(\frac{\pi}{2}) = -1$.

Q:4 (12 points) Solve the following Volterra integral equation $f(t)+\frac{8}{3}\int_0^t f(\tau)(\tau-t)^3 d\tau = 1+t$.

Q:5 (10 points) Solve the following initial value problem using Laplace transform

$y'' + 6y' + 10y = \delta(t - 2\pi)$ with $y(0) = 1$, $y'(0) = 1$.

Q:6 (10 points) Show that $f_1(x) = x^3$ and $f_2(x) = x^2 + 1$ are orthogonal on $[-1, 1]$. Find values of a and b such that both $f_1(x)$ and $f_2(x)$ are orthogonal to $f_3(x) = ax + bx^2 + x^3$.

Q:7 (14 points) Find the Fourier series of the function $f(x) = \begin{cases} 0 & -\frac{\pi}{2} < x < 0 \\ \cos x & 0 \leq x < \frac{\pi}{2} \end{cases}$.

Q:8 (8+4 points) **(a)** Find the half-range Fourier cosine expansion of $f(x) = \sin 4x, \;\; 0 \leq x < \frac{\pi}{8}$.

(b) Use part **(a)** to show that $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$.