King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Major Exam 1
The Second Semester of 2014-2015 (142)

Time Allowed: 120 Minutes

Name: ______________________ ID#: ______________

Instructor: ______________________ Sec #: ________ Serial #: ________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		12
2		10
3		12
4		12
5		10
6		15
7		14
8		15
Total		100

Q:1 (12 points) Find length of the curve traced by $\vec{r}(t) = e^{3t}\cos(2t)\,\mathbf{i} + e^{3t}\sin(2t)\mathbf{j} + e^{3t}\mathbf{k}$ on the interval $0 \leq t \leq 2\pi$. Also find equation of tangent line to the curve at $t = \pi$.

Q:2 (a) (5 points) Find the directional derivative of $f(x, y, z) = 2xz + 3xy^2 + yz^2$ at $(-1, 1, 2)$ in the direction of $2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$.

(b) (5 points) Find the direction in which f increases most rapidly and the value of maximum rate of change of f at $(2, 1, -3)$.

Q:3 (12 points) Let $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\vec{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ is a constant vector. Show that

(a) $\nabla \times [(\vec{r} \cdot \vec{r})\, \vec{a}] = 2(\vec{r} \times \vec{a})$

(b) $\nabla \cdot [(\vec{r} \cdot \vec{r})\, \vec{a}] = 2(\vec{r} \cdot \vec{a})$

Q:4 (12 points) Determine whether the vector field $\vec{F}(x, y, z) = (2x \sin y + e^{3z})\,\mathbf{i} + (x^2 \cos y)\,\mathbf{j} + (3xe^{3z} + 5)\,\mathbf{k}$ is a conservative field. If so, find the potential function $\phi(x, y, z)$ for $\vec{F}$.

Q:5 (10 points) Use Green's theorem to evaluate the line integral

$\oint_C (2x^3 - 2y^3)dx + (2x^3 - 3e^y)dy,$

$C = C_1 \bigcup C_2$, where C_1 is a positively oriented circle $x^2 + y^2 = 9$ and C_2 is a negatively oriented circle $x^2 + y^2 = 4$.

Q:6 (15 points) Find the surface area of the portions of the sphere $x^2 + y^2 + z^2 = 16$ that are within the cylinder $x^2 + y^2 = 4y$.

Q:7 (14 points) Use Stokes' theorem to evaluate the integral $\iint_S curl(F) \cdot \hat{n}\, dS$, where

$\vec{F} = \frac{xz}{4}\mathbf{i} + 4xy\mathbf{j} + 3xyz\mathbf{k}$ and S is the portion of the paraboloid $z = x^2 + 4y^2$

for $0 \leq z \leq 16$ in the first octant.

Q:8 (15 points) Use divergence theorem to evaluate $\iint_S (\vec{F}.\hat{n})\, dS$ where $\vec{F} = 6xz\mathbf{i} + 5y^2\mathbf{j} - 3z^2\mathbf{k}$ and D the region bounded by $z = y$, $z = 4 - y$, $z = 2 - \frac{1}{2}x^2$, $x = 0$ and $z = 0$.