King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 280-01(Term 142)

Final Exam

May 19, 2015

NAME: ....

ID #: . . . .

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Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	

Q1. Let A and B be two $n \times n$ matrices. Let $\lambda \neq 0$ be an eigenvalue of $\mathbf{AB}$. Show that λ is also an eigenvalue of $\mathbf{BA}$

Q2. Let A be a nonsingular matrix and let λ be an eigenvalue of A. Show that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}

Q3. Let A be an $n \times n$ matrix. Show that A and A^T have the same characteristic polynomial and hence the same eigenvalues.

Q4. Let B be a nonsingular matrix and let $M = B^T B$:

(i) Show that M is symmetric.

(ii) Show that M is positive definite.

Q5. Let A be a symmetric positive definite matrix. Show that all the diagonal elements of A are positive.

Q6. (a) State the definition of an **orthogonal matrix**.

(b) If A and B are orthogonal matrices show that AB is also orthogonal.

(c) Show that the determinant of an orthogonal matrix is either 1 or -1.

Q7. (a) Verify that the following is an inner product on $\mathbb{R}^2$:

$$\langle u, v \rangle = x_1y_1 - 2x_1y_2 - 2x_2y_1 + 5x_2y_2$$

where

$$u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \ v = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

(b) Consider the vectors

$$u = \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \ v = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

Find $\| u \|$, $\| v \|$, and the angle θ between u and v.

Q8. Let

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}.$$

Factor A into a product XDX^{-1}, where D is diagonal.

Q9. Let V be the vector space of all 2×2 matrices over $\mathbb{R}$. Let W consists of all matrices A for which $A^2 = A$. Show that W is not a subspace of V.

Q10. Let $Y = \text{Span}(u)$ be a subspace of $\mathbb{R}^3$, where $u = (1, -1, 1)^T$. Find a basis for the orthogonal complement of Y.

Q11. Let L be the linear operator on $\mathbb{R}^3$ defined by

$$L(X) = \begin{bmatrix} x_1 + 2x_2 - x_3 \\ x_2 + x_3 \\ x_1 + x_2 - 2x_3 \end{bmatrix}$$

(a) Find the standard matrix representation of L.

(b) Find the kernal of L.

Q12. Given the basis

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$$

for $\mathbb{R}^3$, use the Gram-Schmidt process to obtain an orthonormal basis.