King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 280-01(Term 142)
Exam II
April 18, 2015

NAME: ....

ID #: . . . .

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Question	Points	Score
1	10	
2	12	
3	12	
4	10	
5	10	
6	12	
7	12	
8	12	
9	10	
Total	100	

Q1. Let W be a subspace of $\mathbb{R}^4$ spanned by the vectors:

$$\begin{bmatrix} 1 \\ 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

Find a basis for W and determine the dimension of W.

Q2. Let T be a mapping from $\mathbb{R}^2$ onto $\mathbb{R}^3$ defined by:

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3x_1 + 2x_2 \\ x_1 \\ -x_1 + 4x_2 \end{bmatrix}$$

(a) Show that T is a linear transformation.
(b) Find a matrix A such that $T(x) = Ax$ for every x in $\mathbb{R}^2$

Q3. (a) Define the rank of a matrix A.

(b)Let

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 1 \\ 3 & 1 & -2 \\ -1 & 0 & 4 \end{bmatrix}$$

determine the following:
i) dimension of the column space of A
ii) dimension of the Row space of A
iii) rank of A
iv) dimension of the null space of A
v) dimension of the null space of A^T

Q4. (a) What Does it mean to say that two $n \times n$ matrices are similar?

(b) If A and B are similar, show that $2A^3 + A - 3I$ and $2B^3 + B - 3I$ are similar.

Q5. Find all values of h so that the set $\{u_1, u_2, u_3\}$ form a basis for $\mathbb{R}^3$, where

$$u_1 = \begin{bmatrix} 4 \\ 4 \\ h \end{bmatrix}, \; u_2 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \; u_3 = \begin{bmatrix} 6 \\ h \\ 6 \end{bmatrix}$$

Justify your answer.

Q6. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear operator and let A be the standard matrix representation of L. where

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Find the matrix B representing T with respect to $\{v_1, v_2\}$, where

$$v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \; v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Q7. Let

$$A = \begin{bmatrix} 1 & 3 & 5 & 9 \\ 2 & 4 & 6 & 7 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

(a) Find a basis for the Null space of A.

(b) Find a basis for the column space of A.

Q8. Let $V = [1, x, x^2]$ and $U = [1, 1 + x, (1 + x)^2]$ be two ordered bases for P_3.

(a) Find the transition matrix from U to V.

(b) Find the transition matrix from V to U.

(c) Use the matrix obtained in (b) above to find the coordinates of $P(x) = a + bx + cx^2$.

Q9. Let V and W be two vector spaces, and let $L : V \longrightarrow W$ be a linear transformation. What is the definition of the kernel of L. Is it a subspace? if yes, of which space?