King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 280-01(Term 142)
Exam I

NAME: ....

ID #: . . . .

SHOW ALL WORK FOR FULL CREDIT

Question	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Q1. Use the Gauss-Jordan reduction method to solve the following homogeneous system:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ x_1 + x_4 &= 0 \\ x_1 + 2x_2 + x_3 &= 0 \end{aligned}$$

Q2. determine which of the following two matrices is nonsingular (explain why) and then find its inverse using the **adjoint** method:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

Q3. Let A be an $m \times n$ matrix. Prove that the diagonal entries of AA^T are nonnegative.

Q4. Let A be a 3×3 nonsingular matrix with $det(A) = 4$. Find the value of $det(3A^{-1})$.

Q5. Let S be a subset of $\mathbb{R}^3$ such that

$$S = \{(a,\ b,\ c)^T |\ \ c = 2a + b\}$$

Show that S is a subspace of $\mathbb{R}^3$

Q6. Determine whether the following are subspaces of $\mathbb{R}^{n\times n}$ and explain why:

a) The set of all $n \times n$ upper triangular matrices.

b) The set of all $n \times n$ singular matrices.

c) The set of all $n \times n$ symmetric matrices.

Q7. (a) State the definition of what it means for a matrix A to be invertible.

(b) Using the definition you gave in (a) to prove that if A is an $n \times n$ invertible matrix then for any vector $b \in \mathbb{R}^n$ the equation $Ax = b$ has a solution and that this solution is unique.

Q8. Let u be a solution of $Ax = b$ and let v be a solution of $Ax = 0$ where A is $m \times n$ matrix and $b \in \mathbb{R}^m$. Show that $u + v$ is a solution of $Ax = b$.

Q9. Let A be an $n \times n$ matrix such that $A^2 = 0$. Show that $(I - A)$ is nonsingular.
(Hint: find the product $(I - A)(I + A)$)