KFUPM **Semester 142**

Dept. Math. &Stat. **A.Y:2014/2015**

TEST 2

(To be submitted by Tuesday, May 05, 2015)

Name : **ID :**

Exercise 1: Show that there is no $a, b \in \mathbf{Z}$ such that $a^2 - 4b = 2$.

Exercise 2: Show that $x = \sqrt{2}$ and $y = \log_2 9$ are irrational numbers. What about x^y?

Exercise 3: Let F_n be the sequence defined recursively by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ (Fibonacci sequence).

Using Mathematical Induction, show the following formulas:

i)

$$F_{n+1}^2 - F_{n+1}F_n - F_n^2 = (-1)^n.$$

ii)

$$F_1 + F_2 + F_3 + F_4 + \ldots + F_n = F_{n+2} - 1.$$

iii)

$$\sum_{k=1}^{n} F_k^2 = F_n F_{n+1}.$$

iv)

$$F_1 + F_3 + F_5 + F_7 + \ldots + F_{2n-1} = F_{2n}.$$

v)

$$F_2 + F_4 + F_6 + F_8 + \ldots + F_{2n} =$$

$$F_{2n+1} - 1.$$