KFUPM **Semester 142**

Dept. Math. &Stat. **A.Y:2014/2015**

TEST 1

Name : **ID :**

Exercise 1:

Let P and Q be statements. Which of the following implies that $P \vee Q$ is false?
(a) $(\sim P) \vee (\sim Q)$ is false. (b) $(\sim P) \vee Q$ is true.
(c) $(\sim P) \wedge (\sim Q)$ is true. (d) $Q \Rightarrow P$ is true. (e) $P \wedge Q$ is false.

Exercise 2:

For statements P and Q, show that $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ is a tautology. Then state $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ in words. (This is an important logical argument form, called **modus ponens**.)

Exercise 3

For statements P, Q and R, show that $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$ is a tautology. Then state this compound statement in words. (This is another important logical argument form, called **syllogism**.)

Exercise 4:

Let R and S be compound statements involving the same component statements. If R is a tautology and S is a contradiction, then what can be said of the following?
(a) $R \vee S$ (b) $R \wedge S$ (c) $R \Rightarrow S$ (d) $S \Rightarrow R$.

Exercise 5:

Using algebraic properties of logical equivalence;

(a) For statements P, Q and R, show that

$$((P \wedge Q) \Rightarrow R) \equiv ((P \wedge (\sim R)) \Rightarrow (\sim Q)).$$

(b) For statements P, Q and R, show that

$$((P \wedge Q) \Rightarrow R) \equiv ((Q \wedge (\sim R)) \Rightarrow (\sim P)).$$

Exercise 6:

Assign propositional variables to come up with the general argument, and then show that the argument is valid. What rules of inference are used?

Argument:

If I drink coffee, then I will get a lot of work done.
If I don't drink coffee, then I am sleepy.
If I am sleepy, then I am grumpy.
Therefore if I don't get a lot of work done, then I am grumpy.

Exercise 7:

An island contains two types of people, knights and knaves. Knights always tell the truth, and knaves always lie. You go to the island, and two people approach you.

Person A says: B is a knight.

Person B says: A and I are of opposite type.

What type are A and B?