# KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 232: FINAL, SEMESTER (142), MAY 19, 2015

**12:30–15:30 pm**

Name : ......................................... ........... ...........

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| **Exercise** | **Points** |
|---|---|
| 1 | : **10** |
| 2 | : **10** |
| 3 | : **14** |
| 4 | : **10** |
| 5 | : **8** |
| 6 | : **10** |
| 7 | : **14** |
| 8 | : **14** |
| 9 | : **14** |
| 10 | : **14** |
| 11 | : **14** |
| 12 | : **8** |
| Total | : **140** |

**Exercise 1** (10 pts)**.** Give a propositional form of the following argument, and show that it is valid:

If we choose nuclear power, then we increase the risk of a nuclear accident.

If we choose conventional power, then we add to the greenhouse effect.

We choose either nuclear power or conventional power.

Therefore, we either increase the risk of a nuclear accident or add to the greenhouse effect.

**Exercise 2** (10 pts)**.** Describe all the partitions of the set $A = \{a, b, c, d\}$.

**Exercise 3** (14 pts)**.** Let $a, b \in \mathbb{C}$. Using mathematical induction, show that for all $n \in \mathbb{N}$,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k.$$

Use the previous formula to evaluate the following sums:

(*i*) $\sum_{k=0}^{n} \binom{n}{k} 2^{2k}$

(*ii*) $\sum_{k=1}^{2n} \binom{2n}{k} (-1)^k 3^{2k-1}$

(*iii*) $\sum_{k=0}^{n} \binom{n}{k} \cos(k\frac{\pi}{2})$

**Exercise 4** (10 pts)**.** Find all integers $x, y$ such that

$$26x + 7y = 2.$$

**Exercise 5** (8 pts)**.** Give the prime factorizations of $x = 67500$ and $y = 3150$, then evaluate $d = gcd(x, y)$. Find a particular couple $(u, v) \in \mathbb{Z}^2$ such that $ux + vy = d$.

**Exercise 6** (10 pts)**.** Let $p$ be a prime number. Show that there is no $a, b \in \mathbb{Z}$ such that

$$a^2 - p^2 b = p.$$

**Exercise 7** (14 pts)**.** Let $p, q$ be distinct prime numbers.

$(i)$ Show that $x = \sqrt{p}$ and $y = \log_p(q^2)$ are irrational numbers.

$(ii)$ What about $x^y$.

(Recall that, for $a > 0$, $\beta = \log_a(\alpha)$ is equivalent to $a^\beta = \alpha$)

**Exercise 8** (14 pts)**.** Prove that the function $f : \mathbb{R} \setminus \{2\} \longrightarrow \mathbb{R} \setminus \{4\}$ defined by $f(x) = \dfrac{4x+1}{x-2}$ is bijective. Find its inverse function $f^{-1}$.

**Exercise 9** (14 pts)**.** Show that the function $f : \mathbb{N} \longrightarrow \mathbb{Z}$ defined by $f(n) = \dfrac{(-1)^n(2n-1)+1}{4}$ is a bijection.

**Exercise 10** (14 pts)**.** Let $a < b$ in $\mathbb{R}$. Show that $|(a,b)| = |[a,b]|$.

**Exercise 11** (14 pts)**.** For a positive integer $n$, we denote by

$$U(n) := \{\bar{i} \in \mathbb{Z}_n : i \text{ has an inverse modulo } n\}$$

List all the elements of $U(10)$ and give the table of multiplication on $U(10)$.

**Exercise 12** (8 pts)**.** Show that for $n \geq 3$, $(S_n, \circ)$ is not commutative.