KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 232: EXAM II, SEMESTER (142), MAY 05, 2015

8:00–10:00 pm

Name : ....

ID : .....

Exercise	Points
1	: **8**
2	: **8**
3	: **8**
4	: **8**
5	: **8**
6	: **14**
7	: **14**
8	: **12**
Total	: **80**

Exercise 1 (8 pts)**.** Describe all the equivalence relations on the set $A = \{a, b, c\}$.

Exercise 2 (8 pts)**.** Define a relation R on $\mathbb{Z}$ as xRy if and only if $3x - 5y$ is even. Show that R is an equivalence relation. Describe its equivalence classes.

Exercise 3 (8 pts)**.** Write the addition and multiplication tables for $\mathbb{Z}_6$.

Exercise 4 (8 pts)**.** Suppose $[a], [b] \in \mathbb{Z}_n$, and $[a] = [a']$ and $[b] = [b']$. Mohammed adds $[a]$ and $[b]$ as $[a]+[b] = [a+b]$, he also writes the multiplication as $[a]\times[b] = [ab]$. Ali adds them as $[a']+[b'] = [a'+b']$; for the multiplication he writes $[a']\times[b'] = [a'b']$. Show that their answers $[a+b]$ and $[a'+b']$ are the same, and the answers $[ab]$ and $[a'b']$ are the same.

Exercise 5 (8 pts)**.**

(*i*) Consider the set $f = \{(x, y) \in \mathbb{Z} : x + 7y = 11\}$. Is this a function from $\mathbb{Z}$ to $\mathbb{Z}$? Explain.

(*ii*) Consider the set $f = \{(x^2, x) : x \in \mathbb{Z}\}$. Is this a function from $\mathbb{Z}$ to $\mathbb{Z}$? Explain.

Exercise 6 (14 pts)**.** Consider the function $h : \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Q}$ defined by $h(m, n) = \dfrac{m}{|n| + 1}$. Determine whether this is injective and whether it is surjective.

Exercise 7 (14 pts)**.** Prove that the function $f : \mathbb{R} \setminus \{2\} \longrightarrow \mathbb{R} \setminus \{5\}$ defined by $f(x) = \dfrac{5x+1}{x-2}$ is bijective. Find its inverse function f^{-1}.

Exercise 8 (12 pts)**.** Let $f : A \longrightarrow B$ be a function. Recall that if D is a subset of B and C is a subset of A, then

$$f(C) = \{f(x) : x \in C\} \text{ and } f^{-1}(D) = \{x \in A : f(x) \in D\}.$$

Show that that f is injective if and only if $C = f^{-1}(f(C))$, for every $C \subseteq A$.