KFUPM, DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 232: EXAM I, SEMESTER (142), MARCH 15, 2015

9:00–11:00 pm

Name : ....

ID : .....

Exercise	**Points**
1	: **10**
2	: **10**
3	: **4**
4	: **4**
5	: **4**
6	: **10**
7	: **10**
8	: **4**
9	: **10**
10	: **10**
11	: **4**
Total	: **80**

Exercise 1. Write the propositional form of the following argument. Then decide whether it is valid or invalid.

Argument:
Either the university budget will continue to increase or the quality of its library holdings will be undermined.
If the university budget increases, the opportunities for students will be better.
If the quality of its library holdings is undermined, the great care will be needed to protect its reputation.
Therefore, either the opportunities for students will be better or great care will be needed to protect the university's reputation.

Exercise 2. Write the propositional form of the following argument. Then decide whether it is valid or invalid.

Argument:
If John does not practice his singing, he will hinder the work of the choir director.
If John hinders the work of the choir director, he should not be allowed to continue as a member of the choir.
John does not practice his singing.
Therefore, John should not be allowed to continue as a member of the choir.

Exercise 3. Let P, Q be two propositions. Give the truth table of $(P \vee Q) \wedge (\overline{P \wedge Q})$.

Exercise 4. Let P, Q be two propositions. Show that

$$(P \Longrightarrow Q) \vee (P \Longrightarrow \overline{Q})$$

is a tautology.

Exercise 5. Let P, Q be two propositions. Show that $(P \Longrightarrow (P \wedge Q))$ and $P \Longrightarrow Q$ are logically equivalent.

Exercise 6. Prove by contrapositive: Let $x \in \mathbb{Z}$. If $x^2 - 4x + 3$ is even, then x is odd.

Exercise 7. Let P, Q, R be propositions. By using a truth table, show that the proposition

$$[(P \vee Q) \wedge (\overline{P} \vee R)] \Longrightarrow (Q \vee R)$$

is a tautology

Exercise 8. Show that for all $x, y \in \mathbb{R}$,

$$||x| - |y|| \leq |x - y|.$$

Exercise 9. Show that for all $n \in \mathbb{Z}$, if n is not divisible by 3, then n^2+2 is divisible by 3.

Exercise 10. Show that for all $x, y \in \mathbb{R}$, the following inequality holds

$$\frac{1}{4}x^2 + y^2 \geq xy.$$

Exercise 11. For each of the following quantified statements, decide whether it is true or false:

(1) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x^2 - y^2 + |xy| \geq 1.$

(2) $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}, x^2 - y^2 + |xy| \geq 1.$

(3) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x^2 - y^2 + |xy| \geq 1.$

(4) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x^2 - y^2 + |xy| \geq 1.$