

Instructions: Show Your Work!

1. (4 pts) Consider the differential equation

$$(\sin x) y'' + y' + 5y = 0. \quad (\text{DE}^*)$$

- (a) Explain why (DE*) has two solutions of the form

$$\sum_{n=0}^{\infty} c_n \left(x - \frac{\pi}{2}\right)^n.$$

- (b) Find the minimum radius of convergence of the power series solutions in part (a).

2. (3 pts) Classify the singular points of the following differential equation

$$x^3 (x^2 - 25) (x - 2)^2 y'' + 3x (x - 2) y' + 7 (x - 5) y = 0.$$

3. (3 pts) Consider the differential equation

$$2xy'' - y' + 2y = 0,$$

Find the indicial roots of the singularity at the regular singular point $x = 0$.